6.5 Low Noise Amplifier Design

Use constant gain circles and constant noise figure circles to determine best design.

The noise figure of a two-port amplifier can be written as,

\[ F = F_{\text{min}} + \frac{R_N}{G_s} |Y_S - Y_{\text{opt}}|^2 \]

where \( Y_S = G_S + jB_S = \text{source admittance} \)

(which would include matching)

\[ Y_{\text{opt}} = \text{optimum source admittance for minimum noise figure} \]

\[ F_{\text{min}} = \text{min. noise figure} \]

\[ R_N = \text{equivalent noise resistance of transistor} \]

\( G_s = \text{real part of source resistance} \)

Now, \( Y_S \) and \( Y_{\text{opt}} \) can be written in terms of reflection coefficients.

\[ Y_S = \frac{1}{Z_0} \frac{1}{1 + \eta_S}, \quad Y_{\text{opt}} = \frac{1}{Z_0} \frac{1}{1 + \eta_{\text{opt}}} \]

\[ |Y_S - Y_{\text{opt}}|^2 = \frac{4}{Z_0^2} \frac{|\eta_S - \eta_{\text{opt}}|^2}{(1 + \eta_S)^2 (1 + \eta_{\text{opt}})^2} \]

\[ G_S = \Re \{ Y_S \} = \frac{1}{Z_0} \frac{1 - |\eta_S|^2}{(1 + |\eta_S|^2)} \]

Substituting gives

\[ F = F_{\text{min}} + \frac{4R_N}{Z_0} \frac{|\eta_S - \eta_{\text{opt}}|^2}{(1 - |\eta_S|^2)^2 (1 + |\eta_{\text{opt}}|^2)^2} \]
To get equations for noise figure, define the noise figure parameter:

\[ N = \frac{\left| \frac{\eta}{\eta_{Opt}} \right|^2}{\left| 1 - \eta_{Opt}^2 \right|^2} = \frac{F - F_{Min}}{\ln \left( \frac{2N}{R} \right)} \left( 1 + \eta_{Opt}^2 \right)^2 \]

After some algebra,

\[ \left| \frac{\eta}{N+1} - \frac{\eta_{Opt}}{N+1} \right| = \frac{\sqrt{N(N+1) - \left| \eta_{Opt} \right|^2}}{N+1} \]

Circle of constant noise figure

\[
\begin{align*}
C_F &= \frac{\eta_{Opt}}{N+1} \\
R_F &= \frac{\sqrt{N(N+1) - \left| \eta_{Opt} \right|^2}}{N+1}
\end{align*}
\]

(See Example 6.5)

This has been a short introduction to LNA design. For further reading it is suggested that you read "Practical Considerations for Low Noise Amplifier Design," by Tim Ohs, Freescale Semiconductor White Paper.
Example 6.5  GaAs FET, $Z_0 = 50 \Omega$, $f = 4$ GHz.

\[ S_{11} = 0.60 \angle -60^\circ \]
\[ S_{21} = 1.9 \angle 81^\circ \text{ assume} \]
\[ S_{12} = 0.05 \angle 26^\circ \rightarrow 0 \]
\[ S_{22} = 0.5 \angle -60^\circ \]

Noise parameters:
\[ F_{\text{min}} = 1.6 \text{ dB} \]
\[ R_{\text{opt}} = 0.62 \Omega, \quad R_N = 200 \Omega \]

Design for a 3 dB noise figure.

Determine the error resulting from the unilateral assumption.

Unilateral figure of merit
\[
U = \frac{|S_{11}||S_{21}| |S_{21}| |S_{12}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)} = 0.059
\]

\[
\frac{1}{(1+u)^2} < \frac{G_T}{G_{TM}} < \frac{1}{(1-u)^2}
\]

\[ 0.81 < \frac{G_T}{G_{TM}} < 1.13 \]

\[ -0.5 \text{ dB} < G_{T} \rightarrow G_{TM} < 0.53 \text{ dB} \]

\[ \longrightarrow \pm 0.5 \text{ dB error in gain due to approximation.} \]

Next find $G_T, R_T$ for the noise circles.

\[ N = 0.0986 \]

\[ (\delta \text{ dB}) \]

\[ G_T = 0.56 \angle 100^\circ, \quad R_T = 0.24 \]

Next calculate source constant gain circles:

<table>
<thead>
<tr>
<th>$G_S$ (dB)</th>
<th>$G_S$</th>
<th>$C_S$</th>
<th>$R_S$</th>
<th>$S_{11}^*$ = 0.6 $\angle 60^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.805</td>
<td>0.52 $\angle 60^\circ$</td>
<td>0.800</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>0.901</td>
<td>0.56 $\angle 60^\circ$</td>
<td>0.205</td>
<td></td>
</tr>
<tr>
<td>1.7</td>
<td>0.946</td>
<td>0.58 $\angle 60^\circ$</td>
<td>0.150</td>
<td></td>
</tr>
</tbody>
</table>
FIGURE 6.14 Circuit design for the amplifier of Example 6.5. (a) Constant gain and noise figure circles, (b) RF circuit.

several input section constant gain circles. From (6.54) we compute the following data:

<table>
<thead>
<tr>
<th>$G_S$ (dB)</th>
<th>$g_S$</th>
<th>$C_S$</th>
<th>$R_S$</th>
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</tbody>
</table>
Optimum solution is,

\[ P_0 = 0.53 \leq 75^\circ \]

\[ G_5 = 1.7 \text{ dB and } F = 2.0 \text{ dB.} \]

Then

\[ G_L = \frac{1}{1 - |s_{22}|^2} = 1.25 \text{ dB} \]

\[ G_0 = |s_{21}|^2 = 5.58 \text{ dB} \]

And

\[ G_{TU} = G_5 + G_0 + G_L = 8.53 \text{ dB} \]
OTHER TOPICS

- FEEDBACK - USED TO REDUCE NONLINEARITY AND IMPROVE WIDEBAND MATCHING.

- MATCHING - VARIOUS MATCHING TECHNIQUE ARE USED (COMPENSATED MATCHING NETWORKS)

- BIAS NETWORKS

- LAYOUT OPTIMIZATION

- PACKAGE PARASITICS

- TEMP. VARIATION

- ESD

- POWER DISSIPATION