6.4 AMPLIFIER DESIGN USING S-PARAMETERS

DESIGN STEPS

1) CHOOSE ACTIVE DEVICE(S)
2) SINGLE OR MULTISTAGE OR CASCODE DESIGN (CLASS A- F)
3) IDENTIFY STABLE REGION OF OPERATION
4) DESIGN INPUT AND OUTPUT MATCHING NETWORKS FOR GAIN AND/OR NOISE FIGURE
5) DESIGN BIAS NETWORK
6) ADD BIAS/TEMPERATURE STABILITY
7) ITERATE TO IMPROVE BANDWIDTH

— BE SURE TO INCLUDE PARASITICS —

DESIGN FOR MAXIMUM GAIN

MAXIMUM POWER TRANSFER OCCURS WHEN,

\[ P_{\text{in}} = P_{s}^* \]

AND

\[ P_{\text{out}} = P_{L}^* \]

THIS WILL GIVE MAX. TRANSDUCER GAIN,

\[ G_{\text{max}} = \left( \frac{1}{1-|s_{21}|^2} \right) |s_{21}|^2 \left| \frac{1-|p_{L}|^2}{1-|s_{22}|^2 |p_{2}|^2} \right| \]

THE INPUT AND OUTPUT SECTIONS SHOULD BE MATCHED SIMULTANEOUSLY. PRODUCING

\[ P_{s} = \frac{B_1 - \sqrt{B_1^2 - 4|c_1|^2}}{2c_1} \]

AND

\[ P_{L} = \frac{B_2 - \sqrt{B_2^2 - 4|c_2|^2}}{2c_2} \]

WHERE

\[ B_1 = 1 + |s_{11}|^2 - |s_{22}|^2 - |\Delta|^2 \]

\[ B_2 = 1 + |s_{22}|^2 - |s_{11}|^2 - |\Delta|^2 \]

\[ c_1 = s_{11} - s_{22}^* s_{22} \]

\[ c_2 = s_{22} - s_{22}^* s_{22} \]

\[ |\Delta|^2 = |\Delta|^2 \]

NEG. SIGN ONLY
MAXIMUM STABLE GAIN

If the transistor is unconditionally stable,

\[ G_{\text{max}} = \frac{|S_{21}|}{|S_{12}|} (k - \sqrt{k^2 - 1}) \]


Or

\[ G_{\text{MSG}} = \frac{|S_{21}|}{|S_{12}|} \]

Max. Stable Gain.

(See Example 6.3)

CONSTANT GAIN CIRCLES AND DESIGN FOR SPECIFIED GAIN

Sometimes trade-off with other parameters must be considered so max. gain is not always used. You can plot constant gain circles on the Smith chart.

Unilateral case shown in book!

\[ \frac{G_T}{G_{T_u}} = \text{Error in transducer gain caused by approximating } |S_{12}| \text{ as zero} \]

This ratio is bounded by:

\[ \frac{1}{(1+u)^2} < \frac{G_T}{G_{T_u}} < \frac{1}{(1-u)^2} \]

Where \( u \) is, unilateral figure of merit.
EXAMPLE 6.3

CONJUGATE MATCHED AMPLIFIER DESIGN

DESIGN FOR MAX. GAIN, \( f = 4.0 \, \text{GHz} \), USE SINGLE Stub MATCHING.
CALCULATE AND PLOT INPUT RETURN LOSS AND GAIN FOR 3-5 GHz.
\( Z_0 = 50 \, \Omega \)

\[
\begin{array}{c|c|c|c|c|c}
\hline
f (\text{GHz}) & S_{11} & S_{21} & S_{12} & S_{22} \\
\hline
3 & 0.8 \angle 189^\circ & 2.86 \angle 99^\circ & 0.03 \angle 156^\circ & 0.76 \angle -41^\circ \\
4 & 0.72 \angle -116^\circ & 2.6 \angle 26^\circ & 0.03 \angle 155^\circ & 0.73 \angle -84^\circ \\
5 & 0.66 \angle -142^\circ & 2.39 \angle 84^\circ & 0.03 \angle 162^\circ & 0.72 \angle -68^\circ \\
\hline
\end{array}
\]

\[ \Delta \text{ AND K CHECK:} \]
\[
\begin{cases}
\Delta = 0.958 \angle -182^\circ & \text{UNCONDITIONALLY STABLE AT 4.0 GHz,} \\
K = 1.19 \angle 5 & \text{K > 1}
\end{cases}
\]

CONJUGATE MATCHING:
\[
P = P^* \text{ IN}, \quad P = P^* \text{ OUT}
\]
\[
P_s = \frac{B_2 \pm \sqrt{B_2^2 - 4/C_21^2}}{2C_1} = 0.872 \angle 123^\circ
\]
\[
P_c = \frac{B_1 \pm \sqrt{B_1^2 - 4/C_22^2}}{2C_2} = 0.876 \angle 61^\circ
\]

FIND GAIN FACTORS
\[
G_s = \frac{1}{1 - |P_s|^2} = 4.17 = 6.20 \, \text{dB}
\]
\[
G_c = |P_c|^2 = 6.76 = 8.30 \, \text{dB}
\]
\[
G_L = \frac{1 - |P_L|^2}{|1 - S_{22}P_L|^2} = 1.67 = 2.22 \, \text{dB}
\]

\[ G_{\text{MAX}} = 6.2 + 8.3 + 2.22 = 16.7 \, \text{dB} \]
\[ Y_s = \frac{1}{Z_s} \]

\[ Y_L = \frac{1}{Z_L} \]

\[ d_s = 0.121\lambda \]

\[ l_s = 0.209\lambda \]

\[ d_L = 0.206\lambda \]

\[ l_L = 0.206\lambda \]
The Complete Smith Chart
Black Magic Design

Input Matching
(Work Backwards)

\[ R_s = 0.872 \, \angle 123^\circ \]

\[ \lambda_s \approx 0.204 \lambda \]

Book says 0.206 \lambda

Need \approx 3.4

\[ \frac{d_s}{\lambda} = 0.121 \lambda \]

Book says 0.120 \lambda

Radially Scaled Parameters

Rotate Towards Generator
On \( \lambda \) Circle

Y_{open} = 0

\[ \frac{0.329 \lambda}{-0.208 \lambda} \]

Radially Scaled Parameters

Toward Load
Toward Generator

Origin
\[ U = \frac{|S_{11}| |S_{12}| |S_{21}| |S_{22}|}{(1 - |S_{11}|^2)(1 - |S_{22}|^2)} \]

For unilateral case,

\[ G_S = \frac{1 - |P_s|^2}{1 - S_{11} P_s^2} \]

\[ G_L = \frac{1 - |P_L|^2}{1 - S_{22} P_L^2} \]

For conjugate matching,

\[ G_{S_{\text{max}}} = \frac{1}{1 - |S_{11}|^2} \]

\[ G_{L_{\text{max}}} = \frac{1}{1 - |S_{22}|^2} \]

The normalized gain factors are:

\[ g_S = \frac{G_S}{G_{S_{\text{max}}}} = \frac{1 - |P_s|^2}{1 - S_{11} P_s^2} \left(1 - |S_{11}|^2\right) ; \quad 0 \leq g_S \leq 1 \]

\[ g_L = \frac{G_L}{G_{L_{\text{max}}}} = \frac{1 - |P_L|^2}{1 - S_{22} P_L^2} \left(1 - |S_{22}|^2\right) ; \quad 0 \leq g_L \leq 1 \]

These represent circles on the Smith chart.

(After some algebra)
\[
\begin{align*}
C_s &= \frac{g_s S_{11}^*}{1 - (1-g_s)|S_{11}|^2} \\
R_s &= \frac{\sqrt{1-g_s} (1 - |S_{11}|^2)}{1 - (1-g_s)|S_{11}|^2}
\end{align*}
\]

\[
\begin{align*}
C_L &= \frac{g_L S_{22}^*}{1 - (1-g_L)|S_{22}|^2} \\
R_L &= \frac{\sqrt{1-g_L} (1 - |S_{22}|^2)}{1 - (1-g_L)|S_{22}|^2}
\end{align*}
\]

Properties of Gain Circles:

1. Each family of circles has centers that lie on a straight line given by the angle of \( S_{11}^* \) or \( S_{22}^* \).

2. When \( g_s \) or \( g_L = 1 \) \( R_s \) or \( R_L = 0 \).

3. When gain = 0 dB the gain circle passes through the center of the Smith chart.

(See Example 6.4)
EXAMPLE 6.4

DESIGN AN AMPLIFIER WITH THE FOLLOWING Specs.

\[ G = 11 \text{ dB} @ f = 4.0 \text{ GHz} \]

PLOT CONSTANT GAIN CIRCLES FOR \( G_5 = 2 \text{ dB}, \, 3 \text{ dB} \)

AND \( G_L = 0 \text{ dB}, \, 1 \text{ dB} \).

CALCULATE AND PLOT INPUT RETURN LOSS AND OVERALL GAIN FOR 3-5 GHz.

GIVEN A FET WITH \( Z_0 = 50 \text{ ohms} \)

<table>
<thead>
<tr>
<th>( f ) (GHz)</th>
<th>( S_{11} )</th>
<th>( S_{21} )</th>
<th>( S_{12} )</th>
<th>( S_{22} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.8 $\angle -90^\circ$</td>
<td>2.8 $\angle 100^\circ$</td>
<td>0</td>
<td>0.66 $\angle -50^\circ$</td>
</tr>
<tr>
<td>4</td>
<td>0.75 $\angle -120^\circ$</td>
<td>2.5 $\angle -80^\circ$</td>
<td>0</td>
<td>0.64 $\angle -70^\circ$</td>
</tr>
<tr>
<td>5</td>
<td>0.71 $\angle -140^\circ$</td>
<td>2.3 $\angle -60^\circ$</td>
<td>0</td>
<td>0.58 $\angle -85^\circ$</td>
</tr>
</tbody>
</table>

SOLUTION

\( S_{12} = 0 \), \( |S_{11}| < 1 \), AND \( |S_{22}| < 1 \) \( \implies \) UNILATERAL UNCONDITIONALLY STABLE.

\[ G_{S_{\text{MAX}}} = \frac{1}{1 - |S_{11}|^2} = 2.29 = 3.6 \text{ dB} \]

\[ G_{L_{\text{MAX}}} = \frac{1}{1 - |S_{22}|^2} = 1.56 = 1.9 \text{ dB} \]

\[ G_0 = |S_{21}|^2 = 6.25 = 8.0 \text{ dB} \]

\[ \implies G_{T_{\text{UMAX}}} = 3.6 + 1.9 + 8.0 = 13.5 \text{ dB} \]

\[ \geq 2.5 \text{ dB} \] MORE THAN THE SPEC.
We can back off the max gain in an attempt to improve the bandwidth.

Calculate

\[ G_s = 3 \text{dB} \quad g_s = 0.875 \quad C_s = 0.706 \quad L = 120^\circ \quad R_s = 0.166 \]
\[ G_s = 2 \text{dB} \quad g_s = 0.691 \quad C_s = 0.627 \quad L = 120^\circ \quad R_s = 0.294 \]
\[ G_l = 1 \text{dB} \quad g_l = 0.806 \quad C_l = 0.520 \quad L = 120^\circ \quad R_l = 0.303 \]
\[ G_l = 0 \text{dB} \quad g_l = 0.640 \quad C_l = 0.440 \quad L = 120^\circ \quad R_l = 0.440 \]
The constant gain circles are shown in Figure 6.13a. We can choose $G_S = 2$ dB and $G_L = 1$ dB, for an overall amplifier gain of 11 dB. Then we select $\Gamma_S$ and $\Gamma_L$ along these circles as shown, to minimize the distance from the center of the chart (this places $\Gamma_S$ and $\Gamma_L$ along the radial lines at 120° and 70°, respectively). Thus,
\( \Gamma_S = 0.33 \angle 120^\circ \) and \( \Gamma_L = 0.22 \angle 70^\circ \), and the matching networks can be designed using shunt stubs as in Example 6.3.

The final amplifier circuit is shown in Figure 6.13b. The frequency response was calculated using CAD software, with interpolation of the given \( S \) parameter data. The results are shown in Figure 6.13c, where it is seen that the desired gain of 11 dB is achieved at 4.0 GHz. The bandwidth over which the gain varies by \( \pm 1 \) dB or less is about 25%, which is considerably better than the gain bandwidth of the maximum gain design of Example 6.3. The return loss, however, is not very good, being only about 5 dB at the design frequency. This is due to the deliberate mismatch introduced into the matching sections to achieve the specified gain.
(3.4.14)

ion of a 14), the
given by $C_{Ri}$ (i.e., the cir-
with $S_{Ri}$, the point actance
ated such
itude of re resist-
f $\Gamma_i$, pro-
uch that
al value
and $f = 1$

Figure 3.4.6 Stable and unstable regions and constant $G_i$ circles for Example 3.4.2.

**Solution.** (a) The input impedance is obtained from the Smith chart at the point $1/S_{11}^* = 0.44\angle120^\circ$ (see Fig. 3.4.6)—namely,

$$Z_{IN} = 50(-0.5 - j0.46) = -25 - j23 \ \Omega$$

The optimum termination for $G_L$ is

$$\Gamma_L = S_{22}^* = 0.6\angle80^\circ$$

The impedance associated with $\Gamma_L$ is obtained from the Smith chart as $Z_L = 50(0.56 + j1.03) = 28 + j51.5 \ \Omega$. 
(i.e., in the \( \Gamma_s \) plane) containing the noise figure circles. Maximum gain and minimum noise figure cannot, in general, be obtained simultaneously. In Fig. 4.3.3, the maximum \( G_s \) gain of 3 dB, obtained with \( \Gamma_s = 0.7 \sqrt{110^\circ} \), results in a noise figure of \( F_i \approx 4 \) dB; and the minimum noise figure \( F_{\text{min}} = 0.8 \) dB, obtained with \( \Gamma_s = 0.6 \sqrt{40^\circ} \), results in a gain \( G_s \approx -1 \) dB.

In a bilateral case the available gain circles and the noise figure circles are drawn in the \( \Gamma_s \) plane, and the trade-offs that can be made...
Design Procedure

It is intended to design a low noise amplifier having 14dB transducer gain, <1dB noise figure at 12 GHz operating frequency between 50 ohms input and output termination. After DC bias point is maintained, input and output impedance matching circuits are determined and results are analyzed using RF Tool of MATLAB software. The proposed circuit structure with impedance matching circuits are given below.

![Circuit Diagram]

The FHX76LP, super low noise HEMT product of Eudyna Devices Inc., is chosen as amplifier. In datasheet of this product, s parameters and noise parameters are given for $I_{ds} = 10mA$ and $V_{ds} = 2V$ conditions. Therefore DC biasing should meet these conditions. Again from datasheet, we can obtain $I_{ds} = 10mA$ and $V_{ds} = 2V$ for approximately $V_{gs} = -0.2V$.

![DRAIN CURRENT vs. DRAIN-SOURCE VOLTAGE]

Gain, Noise and Stability Circles

Both the available gain and the noise figure are functions of the source reflection coefficient, $\Gamma_s$. To select an appropriate $\Gamma_s$ that provides a suitable compromise between gain and noise, we use the `circle` method of the `rfckt.amplifier` object to place the constant available gain and the constant noise figure circles on the Smith chart.

```
>> circle(unmatched_lna, fc, 'Stab', 'In', 'Stab', 'Out', 'Ga', 6:2:14,'NF', 0.4:0.2:2);
>> legend('Location', 'SouthEast')
```

Both S11 and S22 are less than unity. Therefore stable region should contain the center of the smith chart which means we are in stable region. Selected values are given below.

$$\Gamma_s = |0.5513|, 175.8\ deg$$

$$\Gamma_L = \Gamma_{\text{out}}^* = |0.5266|, 155.8\ deg$$

$$Z_s = 0.2881 + 0.0336i$$

$$Z_L = 0.3229 + 0.1920i$$