Noise Figure of Cascaded Components

The first stage usually has the most impact, with less influence for the following stages.

Given:

\[ N_1 = hB T_0 \]

\[ G_1, F_1, T_{e1} \rightarrow N_1 \rightarrow G_2, F_2, T_{e2} \rightarrow N_2 \]

Find the \( F \) for the system \( T_e \)

\[ N_1 = G_1 (hB T_0 B) + G_1 (hT_{e1} B) \]

Due to \( N_1 \)

Due to Component 1

\[ N_0 = G_1 G_2 hB (T_0 + T_{e}) \]

Equivalent Network

\[ N_x = hB T_0 \]

\[ G_1, F, T_e \rightarrow \]

\[ N_0 = G_1 G_2 hB (T_0 + T_{e}) \]

For \( N_2 \), we have:

\[ N_2 = G_2 (N_x) + G_2 (hT_{e2} B) \]

\[ = G_2 (G_1 (hB T_0 B) + G_1 (hT_{e1} B)) + G_2 (hT_{e2} B) \]

\[ = G_2 G_1 hB \left( T_0 + T_{e1} + \frac{T_{e2}}{G_1} \right) \]

Equating \( N_2 \) to \( N_0 \) gives:

\[ T_e = T_{e1} + \frac{T_{e2}}{G_1} \]

\[ (F-1)T_0 = (F-1)T_0 + \frac{(F_2 - 1)T_0}{G_1} \]

Remember:

\[ T_e = (F-1)T_0 \]

\[ F = F_1 + \frac{F_2 - 1}{G_1} \]
GENERALIZING THIS RESULT GIVES:

\[
T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \ldots
\]

\[
F_e = F_{e1} + \frac{(F_{e2}-1)}{G_1} + \frac{(F_{e3}-1)}{G_1 G_2} + \ldots
\]

(SEE EXAMPLE 3.5)

3.7 DYNAMIC RANGE AND INTERMODULATION DISTORTION

GIVEN A NONLINEAR NETWORK

\[ N_i \rightarrow \square \rightarrow N_o \]

LET US EXPAND THE OUTPUT VOLTAGE IN A TAYLOR SERIES:

\[
N_o = a_0 + a_1 N_x + a_2 N_x^2 + a_3 N_x^3 + \ldots
\]

WHERE

\[
a_0 = U_o(0) = \text{DC output}
\]

\[
a_1 = \left. \frac{dU_o}{dN_x} \right|_{N_x=0} = \text{LINEAR OUTPUT}
\]

\[
a_2 = \left. \frac{d^2U_o}{dN_x^2} \right|_{N_x=0} = \text{SQUARED OUTPUT}
\]

\[
\vdots
\]
EXAMPLE 3.5  ANALYSIS OF A WIRELESS RECEIVER

**Given:**

- LNA
- Bandpass Filter
- Mixer

\[ S_{n, N} \]

\[ G = 10 \text{ dB} \]
\[ L = 1 \text{ dB} \]
\[ F = 2 \text{ dB} \]
\[ F = 1 \text{ dB} \]
\[ L = 3 \text{ dB} \]
\[ F = 4 \text{ dB} \]

\[ \text{LET } N_s = kT_a B \text{ WHERE } T_a = 15^\circ K \text{ FIND } N_0 \text{ (dBm)} \]

\[ T_a = 290^\circ K \]

**Assume:**

- \( Z_0 = 50 \Omega \)
- \( B_{IF} = 10 \text{ MHz} \)

**Convert dB to numeric values**

\[ G_{dB} = 10 \log_{10} G \quad \Rightarrow \quad 10 \left[ \frac{G_{dB}}{10} \right] = G \quad \Rightarrow \quad G = 10 \quad \Rightarrow \quad 10 \left[ \frac{10 \text{ dB}}{10} \right] = 10 \]

\[ F_{dB} = 10 \log_{10} F \quad \Rightarrow \quad 10 \left[ \frac{F_{dB}}{10} \right] = F \quad \Rightarrow \quad F = 10 \left[ \frac{3 \text{ dB}}{10} \right] = 1.585 \]

\[ L = \frac{L}{C} \quad L_2 = 1 \text{ dB} \Rightarrow G_2 = -1 \text{ dB} \quad \Rightarrow \quad 10 \left[ \frac{1}{10} \right] = 0.794 \]

\[ \Rightarrow \quad F_2 = 10 \left[ \frac{1}{10} \right] = 1.259 \]

\[ L = \frac{L}{C} \quad L_3 = 3 \text{ dB} \Rightarrow G_3 = -3 \text{ dB} \quad \Rightarrow \quad 10 \left[ \frac{-3}{10} \right] = 0.501 \]

\[ \Rightarrow \quad F_3 = 10 \left[ \frac{3 \text{ dB}}{10} \right] = 2.512 \]
EXAMPLE 3.5

\[ F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} = 1.585 + \frac{0.259}{10} + \frac{1.512}{7.94} \]

\[ F = 1.80 \rightarrow F_{dB} = 10 \log_{10}(1.80) = 2.56 \text{ dB} \]

\[ F = 2.56 \text{ dB} \]

Using \( F \) above we can calculate \( T_e \)

\[ T_e = (F - 1)T_0 = (1.80 - 1)(290) = 232 \text{ K} \]

\[ T_e = 232 \text{ K} \]

Total Gain, \( G \):

\[ G = G_1 \cdot G_2 \cdot G_3 = (10)(0.794)(0.501) = 3.98 \]

\[ G = 3.98 \]

Now we have all the values we need to calculate \( N_0 \).

\[ N_0 = G_1 G_2 G_3 h_B \left( T_e + T_0 \right) = G h_B \left( T_e + T_0 \right) \]

\[ = (3.98)(1.38 \times 10^{-23}) \times 10^7 \times (15 + 232) \]

\[ N_0 = 1.36 \times 10^{-13} \text{ W} \]

\[ N_0 = 10 \log_{10} \left( \frac{1.36 \times 10^{-13}}{1 \times 10^{-3}} \right) \text{ dBm} \]

\[ N_0 = -98.7 \text{ dBm} \]

\[ S_n(W) = \frac{N_0}{2B} = \frac{1.36 \times 10^{-13}}{2 \times 10^7} = 6.8 \times 10^{-21} \text{ W/Hz} \]

\[ S_n(W) = 6.8 \times 10^{-21} \text{ W/Hz} \]
Example 3.5

For SNR of 20 dB = 100 = \( \frac{S_o}{N_0} \)

\[
S_o = \frac{S_i}{G} = \frac{S_o}{N_0} \frac{N_0}{G} = 100 \left( \frac{1.36 \times 10^{-13}}{7.98} \right) = 3.42 \times 10^{-12} \text{ W}
\]

\[
S_i = 10 \log \left( \frac{3.42 \times 10^{-12}}{10^{-3}} \right) = -84.7 \text{ dBm}
\]

\[
S_i = -84.7 \text{ dBm}
\]

\( Z_0 = 50 \Omega \)

\[
S_i = V_i \cdot I_i = \frac{V_i \cdot U_j}{Z_0} = \frac{V_j}{Z_0}
\]

\[
V_j = Z_0 S_i
\]

\[
V_i = \sqrt{Z_0 S_i} = \sqrt{50 (3.42 \times 10^{-12})} = 13.08 \mu \text{V (rms)}
\]

\[
V_i = 13.1 \mu \text{V (rms)}
\]

\[\star\]

\[
N_0 = N_i \cdot F \left( \frac{S_o}{S_i} \right) = N_i \cdot F \cdot G = \frac{k T_{0} B \cdot F \cdot G}{N_i}
\]

\[\underline{\text{Gives the wrong answer!}}\]

\[
F \text{ is defined with } N_j = k T_{0} B
\]

\[\underline{\text{Use noise temperature}}\]

\[\underline{\text{To find noise power to avoid this error!}}\]
GAIN COMPRESSION

APPLY A SINEWAVE TO THE INPUT,

\[ V_x(t) = V_0 \cos(\omega_0 t) \]

USING THE TAYLOR SERIES,

\[ V_0 = a_0 + a_1 V_0 \cos \omega_0 t + a_2 V_0^2 \cos 2\omega_0 t + \ldots \]
\[ = (a_0 + \frac{1}{2} a_2 V_0^2) + (a_1 V_0 + \frac{3}{4} a_3 V_0^3) \cos \omega_0 t + \frac{1}{2} a_2 V_0^2 \cos 2\omega_0 t + \frac{1}{4} a_3 V_0^3 \cos 3\omega_0 t + \ldots \]

VOLTAGE GAIN OF NETWORK

\[ G_V = \frac{V_0^{\omega_0}}{V_x^{\omega_0}} = \frac{a_1 V_0 + \frac{3}{4} a_3 V_0^2}{V_0} = a_1 + \frac{3}{4} a_3 V_0^2 \]

Based on 1st three terms

\[ G_V = a_1 + \frac{3}{4} a_3 V_0^2 \quad (a_3 \text{ is usually neg.}) \]

If neg then \( G_V < a_1 \) OR \( G_V < \frac{\partial V_0}{\partial V_0} \)

\[ P_{out} \text{ (dBm)} \]

\[ P_{in} \text{ (dBm)} \]

**GAIN COMPRESSION OR SATURATION**

\[ P_{out} = 1 \text{ dB compression point} \]

\[ P_{in} \text{ (referred to input)} \]

\[ P_{out} \text{ (referred to output)} \]

Slope 1
INTERMODULATION DISTORTION

Consider a two-tone signal,

\[ V_c = V_0 \cos \omega_1 t + \cos \omega_2 t \]

The Taylor series is,

\[
V_0 = a_0 + o_1 V_0 \cos \omega_1 t + a_1 V_0 \cos \omega_2 t + \frac{1}{2} a_2 V_0^2 (1 + \cos 2\omega_1 t) \\
+ \frac{1}{2} a_2 V_0^2 (1 + \cos 2\omega_2 t) + a_2 V_0^2 \cos (\omega_1 - \omega_2) t + a_2 V_0^2 \cos (\omega_1 + \omega_2) t \\
+ a_3 V_0^3 \left( \frac{3}{4} \cos \omega_1 t + \frac{1}{4} \cos 3\omega_1 t \right) + a_3 V_0^3 \left( \frac{3}{4} \cos \omega_2 t + \frac{1}{4} \cos 3\omega_2 t \right) \\
+ \ldots
\]

Lots of harmonics of the form

\[ m \omega_1 + n \omega_2 \]

WHERE \( m, n = 0, \pm 1, \pm 2, \pm 3, \ldots \)

COMBINATIONS OF THE TWO INPUT FREQ. ARE CALLED
INTERMODULATION PRODUCTS
OF ORDER \( |m| + |n| \)

For example,

\[ 2\omega_1 \] second harmonic of \( \omega_1 \) \( m = 2, n = 0 \) ORDER = 2

\[ 2\omega_2 \] of \( \omega_2 \) ORDER = 2

\[ \omega_1 - \omega_2 \] difference freq. \( m = 1, n = 1 \) ORDER = 2

\[ \omega_1 + \omega_2 \] sum freq. \( m = 1, n = 1 \) ORDER = 2

USEFUL FOR MIXER BUT NOT AMPLIFIER.
Chapter 3: Noise and Distortion in Microwave Systems

**FIGURE 3.28** Third-order intercept diagram for a nonlinear component.
From Figure 3.28 we can find $P_3$.

Let $V_{IP}$ be the input signal voltage at the intercept point,

$$V_{IP} = \sqrt{\frac{V_{i1}}{3a_3}}$$

Then,

$$P_3 = \left. P_{W1} \right|_{V_0 = V_{IP}} = \frac{1}{2} a_1^2 V_{IP}^2 = \frac{2a_1^3}{3a_3}$$

$\text{THIRD-ORDER INTERCEPT POWER REFERRED TO OUTPUT}$

$\text{DYNAMIC RANGE}$

Define,

$$DR_Q = \text{LINEAR DYNAMIC RANGE}$$

$\text{NOISE} \quad \rightarrow \quad P_1$

$$DR_F = \text{SPURIOUS-FREE DYNAMIC RANGE}$$

$\text{NOISE} \quad \rightarrow \quad \text{DISTORTION BECOMES UNACCEPTABLE}$

$$DR_F = \frac{P_{W1}}{P_{2w_1-w_2}} \rightarrow \text{OUTPUT POWER AT W_1}$$

$$P_{2w_1-w_2} = \frac{(P_{W1})^2}{(P_3)^2} \Rightarrow \left[ \begin{array}{c} \text{OR} = \frac{P_{W1}}{P_{2w_1-w_2}} \bigg|_{P_{2w_1-w_2} = N_0} = \left( \frac{P_3}{N_0} \right)^{2/3} \end{array} \right]$$
3.7 Dynamic Range and Intermodulation Distortion

![Graph illustrating dynamic range and spurious free dynamic range.]

**Figure 3.29** Illustrating linear dynamic range and spurious free dynamic range.

\[
DR_F (\text{dB}) = \frac{2}{3} (P_3 - N_0)
\]

*(See Example 3.6)*

**Intercept Point of Cascaded Components**

\[
P_3 = \left( \frac{1}{G_2 \rho_3'} + \frac{1}{\rho_3''} \right)^{-1}
\]
EXAMPLE 3.6

A receiver has

\[ F = 7 \text{ dB} \]

\[ P_1 = 25 \text{ dBm (ref. to output)} \]

\[ G = 40 \text{ dB} \]

\[ P_3 = 35 \text{ dBm (ref. to output)} \]

An antenna with \( T_A = 150 \text{ K} \) is the source for the receiver. \( \text{SNR} = 10 \text{ dB} \) is desired. \( \text{BW} = 100 \text{ MHz} = B \)

Find: \( \text{OR}_L \) and \( \text{OR}_F \).

From Eqn. 3.65

\[ N_0 = G_{\text{AB}} \left( T_A + T_c \right) \]

\[ = G_{\text{AB}} \left( T_A + (F-1)T_0 \right) \]

\[ \uparrow \text{Amplified} \quad \uparrow \text{Internally Generated} \quad \uparrow \text{Input Noise} \\quad \text{Noise}. \]

\[ N_0 = (10^4)(1.38 \times 10^{-23})(10^8) \left[ 150 + (5.91-1)(290) \right] \]

\[ = 1.81 \times 10^{-8} \text{ W/Hz} \]

\[ N_0 = 10^4 \log_{10} \left( \frac{1.81 \times 10^{-5}}{1 \times 10^{-3}} \right) \]

\[ N_0 = -47.4 \text{ dBm} \]

\[ \text{OR}_L = P_1 - N_0 = 25 \text{ dBm} - (-47.4 \text{ dBm}) = 72.4 \text{ dB} \]

\[ \text{OR}_L = 72.4 \text{ dB} \]

From Eqn. 3.109

\[ \text{OR}_F = \frac{2}{3} \left( \frac{P_3 - N_0 - \text{SNR}}{10} \right) \]

\[ = \frac{2}{3} (35 \text{ dBm} - (-47.4 \text{ dBm}) - 10 \text{ dB}) = 48.3 \text{ dB} \]

\[ \text{OR}_F = 48.3 \text{ dB} \]
**Example 3.7: Given**

- $G = 20 \text{ dB}$
- $P_3' = 22 \text{ dBm}$ (ref to output)
- $L = 6 \text{ dB}$
- $P_3'' = 13 \text{ dBm}$ (ref to input)

Find $P_3''$

First we transfer $P_3''$ to refer to the output.

$$P_3'' = P_3'' - L = 13 \text{ dBm} - 6 \text{ dB} = 7 \text{ dBm}$$

The equation for $P_3$ requires multiplication so we first convert to numerical values.

$$P_3' = 22 \text{ dBm} = 10 \log \frac{P_3'}{1 \times 10^{-3} \text{ W}}$$

$$22 = \log \frac{P_3'}{1 \times 10^{-3} \text{ W}}$$

$$10^{22} = \frac{P_3'}{1 \times 10^{-3} \text{ W}}$$

$$P_3' = 158.5 \text{ mW}$$

$$P_3'' = 7 \text{ dBm} = 10 \log \frac{P_3''}{1 \times 10^{-3} \text{ W}}$$

$$10^7 = \frac{P_3''}{1 \times 10^{-3} \text{ W}}$$

$$P_3'' = 5.0 \text{ mW}$$

$$G_2 = -6 \text{ dB} = 10 \log \frac{G_2}{10}$$

$$10^{-0.6} = G_2 \Rightarrow 0.25$$

$$P_3 = \left( \frac{1}{G_2 P_3'} + \frac{1}{P_3''} \right)$$

$$= \left( \frac{1}{0.25 \times 158.5} + \frac{1}{5} \right)^{-1}$$

$$= 4.44 \text{ mW}$$

$$P_3 = 10 \log \frac{\left(4.44 \times 10^3 \text{ mW} \right)}{1 \times 10^{-3} \text{ W}} = 6.47 \text{ dBm}$$
Decibel (dB) operates on unitless parameters.

\[ dB = 10 \log_{10} (x) \]

Since power is not unitless (watts) you can't express power in dB.

However \( \frac{P (\text{w})}{1 (\text{w})} \) is unitless

\[ P (\text{dBw}) = 10 \log_{10} \left( \frac{P_{\text{w}}}{1 \text{w}} \right) \]

(OR) \( \frac{P (\text{w})}{1 (\text{mw})} \) is unitless

\[ P (\text{dBm}) = 10 \log_{10} \left( \frac{P_{\text{w}}}{1 \text{ mw}} \right) \]

Safe way to perform math:
When adding or subtracting with mixed dB, dBm, dBw first convert all powers to watts then perform the math.