Chapter 1: Some background material (Correlation with stochastic processes)
Chapter 2: Wiener Filter Theory
Chapter 3: Optimization using Steepest Descent
Chapter 4: Least Mean Square (LMS) adaptive filters
Chapters 5-9: Heuristic of Least-Squares and Recursive Least-Squares

Outline of the course

The focus of this course are adaptive filters.

Adaptive filter - system (filter) whose structure is alterable or adjustable in such a way that its behaviour or performance (according to some desired criterion) improves through contact with its environment.

Simple example of an adaptive system: Automatic Gain Control (AGC) used in radio receivers:

- Input signal + noise
- AGC adjusts the amplifier so

\[ Y \rightarrow \text{adjustable gain} \rightarrow \text{processing of the constant power signal} \]

- Power measurement
- AGC adjusts the amplifier to

Note: The processing is always performed on the signal within some given power range.
The purpose of the course:

* Explain some basic principles of adaptation (learning)
* Explain basic structures used in adaptive systems design
* Examine opening characteristics and performance of adaptive systems
* Explain some methods for physical realisation of adaptive systems

The types of systems discussed include those designed primarily for adaptive control and digital signal processing applications.

* Why do we use adaptive filters?

* Note: Filter bank is used in broad sense as a description of a system performing a signal processing task.

Consider fixed design scenario:

1. Specifications of an ideal system performance
2. Specifications of the desired signal characteristics (PSD, autocorrelation function, etc.)
3. Specifications of the impairments from the environment (noise, interference, linear and nonlinear distortion, etc.)
4. Preferred filter class (Linear, FIR, IIR, Coarse, etc.)

1 => into design process => results in the fixed filter design that will be optimal with respect to provided inputs.

Problems with fixed design scenario:

* Specifications may not be expressed in terms of filtering requirements
* Signal characteristics may not be known in advance, or may change over time
* Impairments may be unknown in advance - For example, interferers may be present or absent, channel characteristics may change over time, PSD of the noise may be time-varying etc.

Adaptive filters can address all of the above problems faced by fixed design.
Open and Closed-loop Adaptation

There are 2 basic adaptation principles commonly referred to as the open-loop and closed-loop adaptation.

**Open-loop adaptation**

- Input signal
- Other data
- Signal processor
- Algorithm
- Adaptation
- Output signal

In open-loop adaptation, the output signal is not available to the processing algorithm.

* Input signal is recorded.
* Parameters of the environment are measured.
* Parameters of the system (adaptation filter) are adjusted on the basis of two inputs.

**Closed-loop adaptation**

- Input signal
- Other data
- Adaptation
- Algorithm
- Output signal

In closed-loop adaptation, the output signal is used to determine the direction of system adjustment.

* Input signal is recorded.
* Output signal is recorded.
* Other parameters may be measured.
* Decision on the system adjustment is based on both input and output signal.
Closed-loop systems — superior performance and used wherever possible.

The most common configuration of a closed-loop system

- Performance feedback implementation of the adaptive system

Available inputs to adaptive algorithm

\[ x(n) \] - input signal
\[ e(n) \] - error signal (alternative: \( y(n) \) & \( d(n) \))
\[ y(n) \] - output signal
\[ d(n) \] - desired output (or data)

The adaptation of the filter parameters is performed in a manner that minimizes some measure of the error, i.e., \( \sum |e(n)| \), \( n=1,2, \ldots \)

Some applications of adaptive filters:

1. Adaptive equalization
2. Speech coding (linear prediction)
3. Spectra analysis
4. Adaptive noise & interference cancellation
5. Adaptive beamforming
6. System identification
7. Signal separation
8. Impulse modeling, etc.
Stochastic processes & models

* Stochastic/random process - time evolution of a statistical phenomenon according to a probabilistic law
* Most of the signals that we come across are stochastic processes. Examples include: speed signal, television signal, radar signals, computer data, seismic data, and noise.
* In this cause the stochastic processes are discrete - defined in discrete and uniformly spaced instants of time. Such processes may occur naturally (like in radio signals or computer communication) or as a result of sampling.

Consider a sample time series of a stochastic process

\[ u(n), u(n-1), \ldots, u(n-H) \]

To completely specify statistical properties of this sequence one needs to know joint probability density function

\[ f(n) = P(u(n), u(n-1), \ldots, u(n-H)), \quad n = 0, 1, \ldots \]

If \( f(n) \) does not depend on \( n \), the process is stationary, in a strict sense.

Joint probability density function provides full characterization of a random process. In practice \( f(n) \) is usually unknown.

Partial characterization of a discrete time random process:

* In practice, joint probability density function - difficult to determine
* Partial characterization of the process - specification of its first & second moments
Consider a discrete-time stochastic process represented by a finite series:

\[ u(n), u(n-1), \ldots, u(n-N) \]

The first moment of the process is defined as:

\[ y(n) = E[y(n)] = \int_{-\infty}^{\infty} x \cdot P(y(n) = x) \, dx = \text{ensemble average} \]

The second moment of the process is defined as:

\[ r(n,n-k) = E[u(n) \cdot \overline{u(n-k)}], \quad k = 0, 1, \ldots \text{ - double complex conjugates} \]
Second moment of the random process is denoted as autocorrelation.

Second central moment of the process is defined as

\[ c(n,n-k) = E \left[ (u(n) - \mu(n)) \cdot (u(n-k) - \mu(n-k)) \right], \quad k = 0,1, \ldots \]

It is obvious that we have

\[ c(n,n-k) = E \left[ u(n) \cdot u(n-k) - \mu(n) \cdot u(n-k) - \mu(n-k) \cdot u(n) + \mu(n) \cdot \mu(n-k) \right] = \]

\[ = E \left[ u(n) \cdot u(n-k) \right] - \mu(n) \cdot \mu(n-k) = r(n,n-k) - \mu(n) \cdot \mu(n-k). \]

If a discrete-time stochastic process is stationary, the above moments do not depend on \( n \) and hence

\[ \mu(n) = \mu \]

\[ r(n,n-k) = r(k), \quad (1) \]

\[ c(n,n-k) = c(k). \]

Also one notes that for \( k = 0 \)

\[ r(0) = E \left[ u(n) \cdot u(n) \right] = E \left[ u(n) \right]^2 \quad \text{variance value of } u(n) \]

\[ c(0) = E \left[ (u(n) - \mu)(u(n) - \mu) \right] = E \mu^2 \quad \text{standard deviation of } u(n) \]

For practical characterization of a stochastic process we need to know at least \( \mu, r(k) \) or \( \mu \) and \( c(k) \).

Process that satisfies (1) is termed wide sense stationary. Strictly stationary process is wide stationary if \( E \left| u(n) \right|^2 < \infty \), which is ordinarily satisfied by most physical processes.