**Problems 1-6: Rutherford scattering**

1) Das & Ferbel, Ch.1 - Problem 1.1

2) Calculate the differential cross section $d\sigma/d\Omega$ for backscattering of 10 MeV $\alpha$-particles by lead nuclei. Give your answer in barn/sterad (barn/sr).

3) Das & Ferbel, Ch.1 - Problem 1.4

4) Read Das & Ferbel, Ch.1, Sec. 1.5. Do Ch.1 - Problem 1.3.

5) A. Show that the distance of closest approach of the $\alpha$-particle to the nucleus can be written as:

$$r_0 = \frac{b \cos(\frac{\Theta_s}{2})}{1 - \sin(\frac{\Theta_s}{2})},$$

where $\Theta_s$ is the scattering angle and $b$ the impact parameter. Calculate $r_0$ for 5 MeV $\alpha$-particles scattered by gold nuclei under impact parameters of a) 1 fm, b) 10 fm, c) 100 fm, d) 1000 fm. You may want to double-check your answers using the “Rutherford calculator” on the Hyperphysics web site (follow the link on the companion web site).

B. In the derivation of the hyperbolic trajectories encountered in Rutherford scattering, we used the integration result below. Show that this result is indeed correct.

$$\int \frac{-dx}{\sqrt{-x^2 - \frac{2}{a} x + \frac{b}{a^2}}} = \arccos\left(\frac{ax + 1}{\sqrt{b + 1}}\right), \quad \text{where} \quad a = \frac{4\pi e_0 L^2}{q_1 q_2 m}, \quad b = \frac{2EL^2 (4\pi e_0)^2}{(q_1 q_2)^2 m}.$$

C. Realistic total Rutherford cross section: Calculate an approximate total cross section for Rutherford scattering of a 5 MeV $\alpha$-particle by a gold atom taking into account that the charge of the nucleus is being screened by the electrons. Assume that the charge of the nucleus is fully screened if the impact parameter is larger than the average radius of the outermost electron orbits.

6) Das & Ferbel, Ch.1 - Problem 1.6

**Problem 7: Total cross section and Luminosity**

The Large Hadron Collider at CERN will eventually collide bunches of protons with counter-rotating bunches of protons with a frequency of $f = 40$ MHz at each interaction point. The number of protons in a bunch for each beam is $N_1 = N_2 \approx 10^{11}$. The beams have Gaussian profiles with widths $\sigma \approx 20 \mu m$. The total production cross section for producing a pair of top and anti-top quarks at the LHC is about 900 pb (picobarn). How many top/anti-top pairs will be produced at the LHC every day (assuming the collider runs 24 hours a day)? Note: The instantaneous luminosity of a collider with Gaussian beam profile is given by: $L_{\text{inst}} = f \left( \frac{N_1 N_2}{4\pi \sigma^2} \right)$. 
