1. Precise and complete answers are a must for full credit. Show all your work. Calculators are **NOT** allowed.

1. State/ Define the following precisely. ( $5 \times 2 = 10$ )
   (i) Denumerable set
   (ii) surjective (onto) function
   (iii) Completeness axiom
   (iv) Archimedean property of $\mathbb{R}$
   (v) divergent sequence

2. Suppose $S$ is a nonempty subset of $\mathbb{R}$ and $k$ is an upper bound of $S$. Then show that $k$ is the least upper bound of $S$ if and only if for each $\varepsilon > 0$ there exists a $s \in S$ such that $k - \varepsilon < s$. (5 points)

3. Prove that between any two real numbers there exists an irrational number. (3 points)

4. Prove that the sequence $\{a_n\}$ converges to $A$ if and only if $\lim_{n \to \infty} (a_n - A) = 0$. (5 points)

5. Show that the sequence $\sqrt{n}$ diverges.