Problem 1 (25 Points):
A rocket engine uses propellants that produce gases such that c*=1,500 m/s and γ=1.2. The rocket is required to produce a thrust of 100,000 N, with an ideally expanded nozzle operating at a chamber pressure of 100 atm. and one atmosphere ambient pressure. What is the required throat area and mass flow rate to produce this thrust level?

\[ T = \dot{m} U_e = \dot{m} C^* C_T \]

\[ C_T = \sqrt{\frac{2 \gamma^2}{\gamma-1} \left( \frac{2}{\gamma+1} \right)^{(\gamma+1)/(\gamma-1)} \left[ 1 - \left( \frac{P_e}{P_c} \right)^{(\gamma-1)/\gamma} \right]} \]

\[ \frac{5.0471}{1 - \left( \frac{1}{100} \right)^{2/\gamma}} \]

\[ C_T = 1.645 \]

\[ T = \dot{m} \left( \frac{P_c A^*}{\dot{m}} \right) C_T \]

\[ A^* = \frac{T}{P_c C_T} = \frac{100,000 \text{ N}}{(100)(101325)(1.645)} = 0.006 \text{ m}^2 = 60 \text{ cm}^2 \]

\[ U_e = C^* C_T = (1,500 \text{ m/s})(1.645) = 2467.5 \text{ m/s} \]

\[ \dot{m} = \frac{T}{U_e} = \frac{100,000}{2467.5} = 40.5 \text{ Kg/s} \]
Problem 2 (25 Points):
A rocket fired straight up from the Earth has an initial propellant mass equal of 5,000 kg, an exhaust velocity of 3,000 m/s, and initial thrust of 200,000 N. The initial mass of the vehicle is 10,000 kg. What is the velocity of the vehicle at burnout? You may assume constant gravity, ideal expansion throughout the flight profile and neglect aerodynamic drag.

\[ \Delta V = U_e \ln \left( \frac{M_0}{m(t_f)} \right) - g t_b \]

\[ U_e = 3,000 \text{ m/s} \]

\[ T = m U_e = 200,000 \]

\[ m = \frac{T}{U_e} = \frac{200,000}{3} = 66.7 \text{ kg/s} \]

\[ M_0 = 10,000 \text{ kg} \]

\[ \Delta M = (50\%)(10,000) = 5,000 \text{ kg} \]

\[ t_b = \frac{5,000}{66.7} = 75 \text{ seconds} \]

\[ \Delta V(t_b) = 3,000 \ln \left( \frac{10,000}{5,000} \right) - (9.8)(75) \]

\[ \Delta V = V_{final} - v_0 = 1344 \text{ m/s} \]

\[ V_{final} = 1344 \text{ m/s} \]
Problem 3 (25 Points):

- **Part 1:** Using a propellant of molecular weight 15 and combustion chamber temperature of 3,300 K, determine the rocket throat and exhaust areas required for a thrust of 500 kN and an ideal specific impulse of 300 seconds. The ambient pressure is 0.1 MPa, and the specific heat ratio of the propellant is 1.4.

- **Part 2:** How much thrust would this rocket develop if the ambient pressure were changed to 0.03 MPa?

- **Part 3:** How much thrust would be developed by a rocket designed to expand to 0.03 MPa if it had the same stagnation conditions, throat area, and propellant?

\[
\frac{1}{P^*P_0} = \sqrt{\frac{28}{\gamma-1}} \left( \frac{2}{\gamma+1} \right)^{\gamma+1} \left( \gamma - 1 \right)^{\frac{\gamma-1}{\gamma}} \left( 1 - \left( \frac{P_0}{P} \right)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma+1}{\gamma-1}}
\]

\[
T = \sqrt{\frac{28}{\gamma-1} \frac{R T_0}{\gamma} \left[ 1 - \left( \frac{P_0}{P} \right)^{\frac{\gamma-1}{\gamma}} \right]}
\]

\[
(9,813,000) = \sqrt{7 \left( \frac{8315}{24} \right) \left( 3300 \right) \left[ 1 - \left( \frac{0.1}{P_0} \right)^{\frac{\gamma-1}{\gamma}} \right]}
\]

\[
P_0 = 5.190 \text{ MPa}
\]

\[
\frac{500,000}{(25)(5.19 \times 10^6)} = \sqrt{7(1.4)\left( \frac{2}{2.4} \right)^6 \left[ 1 - \left( \frac{0.1}{5.19} \right)^{\frac{\gamma-1}{\gamma}} \right]}
\]

\[
P^* = 0.06466 \text{ m}^2 \Rightarrow
\]

\[
M_e = \left[ (\frac{2}{\gamma-1})^{\frac{\gamma+1}{\gamma}} \right]^{\frac{\gamma+1}{\gamma-1}} = \left[ 5 \left( \frac{5.19}{0.1} \right)^{\frac{\gamma+1}{\gamma-1}} \right] = 3.233
\]

\[
\frac{A_c}{A^*} = \frac{1}{M_e} \left[ \frac{2}{\gamma+1} \left( 1 + \frac{\gamma-1}{\gamma} M_e^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}}
\]

\[
= \frac{1}{3.233} \left[ \frac{2}{2.4} \left( 1 + \frac{3.233^2}{5} \right) \right]^{3} = 5.284
\]

\[
A_e = P^* \left( \frac{A_c}{A^*} \right) = 0.3416 \text{ m}^2
\]
2) with \( P_0 = 0.03 \text{ MPa} \)

\[
T = \dot{m} U_e + (P_e - P_a) A_e \\
= T_0 + (P_e - P_a) A_e \\
= 500 + (0.1 - 0.03) \times 10^2 (0.3416) \\
= 523.2 \text{ kN}
\]

3) \[
T = \dot{m} U_e' = T_0 \frac{U_e'}{U_e} = T_0 \sqrt{1 - \left( \frac{P_e'}{P_o} \right)^{\frac{\gamma - 1}{\gamma}}}
\]

\[
= 500 \sqrt{1 - \left( \frac{0.02}{5.19} \right)^{\frac{\gamma - 1}{\gamma}}}
\]

\[
= 533.7 \text{ kN}
\]
Problem 4 (25 Points):
A toy rocket containing water and compressed air has mass 0.20 kg when empty and can carry 1 kg of water. The initial air pressure within the rocket at launch is 0.35 MPa.

Part 1: What is the minimum nozzle diameter that will permit vertical takeoff?
Part 2: What is the specific impulse under these conditions?
Part 3: Why is the water added?

With air/water contact, assume isothermal expansion from Bernoulli:

\[ U_w = \sqrt{\frac{2}{\rho_w} (P - P_e)} \quad \text{at lift-off} \]

\[ T = m_w U_e = \rho_w U_w^2 A_e = 2(P_e - P_0) A_e \]

1) At lift-off

\[ (M_s + M_w) g = 2(P_e - P_0) A_e \]

\[ (A_e)_{\text{minimum}} = \frac{(M_s + M_w) g}{2(P_e - P_0)} = \frac{(0.21 + 1)(9.81)}{2(0.35 - 0.1)} \]

\[ = 23.74 \text{mm}^2 \]
Minimum nozzle diameter:

\[ d_e = \sqrt[4]{\frac{4}{\pi}} A_e \approx \sqrt[4]{\frac{4}{7}} (2.74) \]

= 5.50 mm

2) \[ I_{sp} = \frac{T}{mg} = \frac{U_e}{g} = \frac{1}{g} \sqrt{\frac{2}{\rho_e} (I - h)} \]

Initially,

\[ I_{sp} = \frac{1}{9.81} \sqrt{\frac{2}{1000} (0.35 - 0.1) \times 10^6} \]

= 2.28 s

3) EXAMINE TOTAL IMPULSE

\[ I = \int Fdt = \int mvdv = \int udmdm = U_{cm} \]

\[ \frac{I_{water}}{I_{air}} \sim \frac{1}{\sqrt{\rho_w \cdot P_w}} \]

More total impulse with water than air.