Solution: The magnetic coupling will be strongest at the point where the wires of
the two loops come closest. When the switch is closed the current in the bottom loop
will start to flow clockwise, which is from left to right in the top portion of the bottom
loop. To oppose this change, a current will momentarily flow in the bottom of the
top loop from right to left. Thus the current in the top loop is momentarily clockwise
when the switch is closed. Similarly, when the switch is opened, the current in the
top loop is momentarily counterclockwise.

Problem 6.2 The loop in Fig. 6-18 (P6.2) is in the x-y plane and \( \mathbf{B} = 2B_0 \sin \omega t \)
with \( B_0 \) positive. What is the direction of \( I \) (\( \Phi \) or \( -\Phi \)) at (a) \( t = 0 \), (b) \( \omega t = \pi/4 \), and
(c) \( \omega t = \pi/2 \)?

Solution: \( I = V_{\text{emf}}/R \). Since the single-turn loop is not moving or changing shape
with time, \( V_{\text{m}} = 0 \) V and \( V_{\text{emf}} = V_{\text{emf}}^{\text{tr}} \). Therefore, from Eq. (6.8),

\[
I = V_{\text{emf}}^{\text{tr}}/R = -\frac{1}{R} \int_{S} \frac{\partial \mathbf{B}}{\partial t} \cdot ds.
\]

If we take the surface normal to be \( +\hat{z} \), then the right hand rule gives positive
flowing current to be in the \( +\hat{\phi} \) direction.

\[
I = -\frac{A}{R} \frac{\partial}{\partial t} B_0 \sin \omega t = -\frac{A B_0 \omega}{R} \cos \omega t \quad (A),
\]
where $A$ is the area of the loop.

(a) $A$, $\omega$ and $R$ are positive quantities. At $t = 0$, $\cos \omega t = 1$ so $I < 0$ and the current is flowing in the $-\hat{\phi}$ direction (so as to produce an induced magnetic field that opposes $B$).

(b) At $\omega t = \pi/4$, $\cos \omega t = \sqrt{2}/2$ so $I < 0$ and the current is still flowing in the $-\hat{\phi}$ direction.

(c) At $\omega t = \pi/2$, $\cos \omega t = 0$ so $I = 0$. There is no current flowing in either direction.

**Problem 6.3** A coil consists of 100 turns of wire wrapped around a square frame of sides 0.25 m. The coil is centered at the origin with each of its sides parallel to the $x$- or $y$-axis. Find the induced emf across the open-circuited ends of the coil if the magnetic field is given by

(a) $B = 210e^{-2t}$ (T),

(b) $B = 210 \cos x \cos 10^3 t$ (T),

(c) $B = 210 \cos x \sin 2y \cos 10^3 t$ (T).

**Solution:** Since the coil is not moving or changing shape, $V'_{\text{emf}} = 0$ V and $V_{\text{emf}} = V''_{\text{emf}}$. From Eq. (6.6),

$$V_{\text{emf}} = -N \frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{s} = -N \frac{d}{dt} \int_{-0.125}^{0.125} \int_{-0.125}^{0.125} \mathbf{B} \cdot (2 \, dx \, dy),$$

where $N = 100$ and the surface normal was chosen to be in the $+\hat{z}$ direction.

(a) For $B = 210e^{-2t}$ (T),

$$V_{\text{emf}} = -100 \frac{d}{dt} (10e^{-2t} (0.25)^2) = 125e^{-2t} \text{ (V)}.$$
Problem 6.4 A stationary conducting loop with internal resistance of 0.5 Ω is placed in a time-varying magnetic field. When the loop is closed, a current of 2.5 A flows through it. What will the current be if the loop is opened to create a small gap and a 2-Ω resistor is connected across its open ends?

Solution: $V_{\text{emf}}$ is independent of the resistance which is in the loop. Therefore, when the loop is intact and the internal resistance is only 0.5 Ω,

$$V_{\text{emf}} = 2.5 \text{ A} \times 0.5 \text{ Ω} = 1.25 \text{ V}.$$  

When the small gap is created, the total resistance in the loop is infinite and the current flow is zero. With a 2-Ω resistor in the gap,

$$I = \frac{V_{\text{emf}}}{(2 \text{ Ω} + 0.5 \text{ Ω})} = \frac{1.25 \text{ V}}{2.5 \text{ Ω}} = 0.5 \text{ A}.$$  

Problem 6.5 A circular-loop TV antenna with 0.01 m² area is in the presence of a uniform-amplitude 300-MHz signal. When oriented for maximum response, the loop develops an emf with a peak value of 20 (mV). What is the peak magnitude of $B$ of the incident wave?

Solution: TV loop antennas have one turn. At maximum orientation, Eq. (6.5) evaluates to $\Phi = \int \mathbf{B} \cdot d\mathbf{s} = \pm BA$ for a loop of area $A$ and a uniform magnetic field with magnitude $B = |\mathbf{B}|$. Since we know the frequency of the field is $f = 300$ MHz, we can express $B$ as $B = B_0 \cos(\omega t + \alpha_0)$ with $\omega = 2\pi \times 300 \times 10^6$ rad/s and $\alpha_0$ an arbitrary reference phase. From Eq. (6.6),

$$V_{\text{emf}} = -N \frac{d\Phi}{dt} = -A \frac{d}{dt} [B_0 \cos(\omega t + \alpha_0)] = AB_0 \omega \sin(\omega t + \alpha_0).$$

$V_{\text{emf}}$ is maximum when $\sin(\omega t + \alpha_0) = 1$. Hence,

$$20 \times 10^{-3} = AB_0 \omega = 10^{-2} \times B_0 \times 6\pi \times 10^8,$$
which yields $B_0 = 1.06$ (nA/m).

**Problem 6.6** The square loop shown in Fig. 6-19 (P6.6) is coplanar with a long, straight wire carrying a current

$$i(t) = 2.5 \cos 2\pi \times 10^4 t \quad \text{(A)}.$$  

(a) Determine the emf induced across a small gap created in the loop.

(b) Determine the direction and magnitude of the current that would flow through a 4-$\Omega$ resistor connected across the gap. The loop has an internal resistance of 1 $\Omega$.

Figure P6.6: Loop coplanar with long wire (Problem 6.6).

**Solution:**

(a) The magnetic field due to the wire is

$$B = \oint \frac{\mu_0 I}{2\pi r} = -\hat{y} \frac{\mu_0 I}{2\pi y},$$

where in the plane of the loop, $\hat{\phi} = -\hat{x}$ and $r = y$. The flux passing through the loop
Problem 6.7  The rectangular conducting loop shown in Fig. 6-20 (P6.7) rotates at 6,000 revolutions per minute in a uniform magnetic flux density given by

\[ B = 0.5 \text{ mT}. \]

Determine the current induced in the loop if its internal resistance is 0.5 \(\Omega\).

Solution:

\[ \Phi = \int B \cdot dS = \int_{\text{rectangle}} \left( -\frac{\mu_0 I}{2\pi y} \right) \cdot [-\pi 10 \text{ cm}] \, dy \]

\[ = \frac{\mu_0 I \times 10^{-1}}{2\pi} \ln \frac{15}{5} \]

\[ = \frac{4\pi 	imes 10^{-7} \times 2.5 \cos(2\pi \times 10^4 t) \times 10^{-1}}{2\pi} \times 1.1 \]

\[ = 0.55 \times 10^{-7} \cos(2\pi \times 10^4 t) \text{ (Wb)}. \]

\[ V_{\text{emf}} = -\frac{d\Phi}{dt} = 0.55 \times 2\pi \times 10^4 \sin(2\pi \times 10^4 t) \times 10^{-7} \]

\[ = 3.45 \times 10^{-3} \sin(2\pi \times 10^4 t) \text{ (V)}. \]

(b)

\[ I_{\text{ind}} = \frac{V_{\text{emf}}}{4 + 1} = \frac{3.45 \times 10^{-3}}{5} \sin(2\pi \times 10^4 t) = 0.69 \sin(2\pi \times 10^4 t) \text{ (mA)}. \]

At \( t = 0 \), \( B \) is a maximum, it points in \( \phi \)-direction, and since it varies as \( \cos(2\pi \times 10^4 t) \), it is decreasing. Hence, the induced current has to be CCW when looking down on the loop, as shown in the figure.
The magnetic field $B$ is created by the wire carrying $I$. Choosing $z$ to coincide with the direction of $I$, Eq. (5.30) gives the external magnetic field of a long wire to be

$$B = \frac{\mu_0 I}{2\pi r}.$$ 

From Eq. (6.24),

$$V_{12} = V_{emf} = \int_2^1 (u \times B) \cdot d\mathbf{l} = \int_{r=0.5}^0 (\phi 6\pi r \times 23 \times 10^{-4}) \cdot \mathbf{r} dr$$

$$= 18\pi \times 10^{-4} \int_{r=0.5}^0 r dr$$

$$= 9\pi \times 10^{-4} r^2 \bigg|_{0.5}^0$$

$$= -9\pi \times 10^{-4} \times 0.25 = -707 \text{ (\mu V)}.$$ 

**Problem 6.10** The loop shown in Fig. 6-22 (P.610) moves away from a wire carrying a current $I_1 = 10$ (A) at a constant velocity $u = 5$ (m/s). If $R = 10$ \Omega and the direction of $I_2$ is as defined in the figure, find $I_2$ as a function of $y_0$, the distance between the wire and the loop. Ignore the internal resistance of the loop.

**Solution:** Assume that the wire carrying current $I_1$ is in the same plane as the loop. The two identical resistors are in series, so $I_2 = V_{emf}/2R$, where the induced voltage is due to motion of the loop and is given by Eq. (6.26):

$$V_{emf} = V_{emf} = \oint_C (u \times B) \cdot d\mathbf{l}.$$ 

The magnetic field $B$ is created by the wire carrying $I_1$. Choosing $z$ to coincide with the direction of $I_1$, Eq. (5.30) gives the external magnetic field of a long wire to be

$$B = \frac{\mu_0 I_1}{2\pi r}.$$
For positive values of $y_0$ in the $y$-$z$ plane, $\mathbf{\phi} = \mathbf{r}$, so

\[ u \times B = \mathbf{\phi}|u| \times B = r|u| \times \frac{\mu_0 I_1}{2\pi r} = \frac{\mu_0 I_1 u}{2\pi r}. \]

Integrating around the four sides of the loop with $dl = 2dz$ and the limits of integration chosen in accordance with the assumed direction of $I_2$, and recognizing that only the two sides without the resistors contribute to $V_{\text{emf}}^m$, we have

\[
V_{\text{emf}}^m = \int_{0}^{0.2} \left( \frac{\mu_0 I_1 u}{2\pi r} \right) r_{r=y_0} \cdot (2dz) + \int_{0}^{0.2} \left( \frac{\mu_0 I_1 u}{2\pi r} \right) r_{r=y_0+0.1} \cdot (2dz)
\]

\[
= 4\pi \times 10^{-7} \times 10 \times 5 \times 0.2 \left( \frac{1}{y_0} - \frac{1}{y_0 + 0.1} \right)
\]

\[
= 2 \times 10^{-6} \left( \frac{1}{y_0} - \frac{1}{y_0 + 0.1} \right) \text{ (V)},
\]

and therefore

\[
I_2 = \frac{V_{\text{emf}}^m}{2R} = 100 \left( \frac{1}{y_0} - \frac{1}{y_0 + 0.1} \right) \text{ (nA)}.
\]

**Problem 6.11** The conducting cylinder shown in Fig. 6-23 (P6.11) rotates about its axis at 1,200 revolutions per minute in a radial field given by

\[ B = f \mathbf{6} \text{ (T)}. \]
CHAPTER 6

Problem 6.13 The circular disk shown in Fig. 6-24 (P6.13) lies in the x-y plane and rotates with uniform angular velocity \( \omega \) about the z-axis. The disk is of radius \( a \) and is present in a uniform magnetic flux density \( B = 2B_0 \). Obtain an expression for the emf induced at the rim relative to the center of the disk.

![Figure P6.11: Rotating cylinder in a magnetic field (Problem 6.11).](image-url)

The cylinder, whose radius is 5 cm and height 10 cm, has sliding contacts at its top and bottom connected to a voltmeter. Determine the induced voltage.

Solution: The surface of the cylinder has velocity \( u \) given by

\[
u = \Phi \omega r = \Phi 2\pi \frac{1,200}{60} \times 5 \times 10^{-2} = \Phi 2\pi \quad \text{(m/s)},
\]

\[
V_{12} = \int_{0}^{L} (u \times B) \cdot dl = \int_{0}^{0.1} (\Phi 2\pi r \times \Phi 6) \cdot 2 \, dz = -3.77 \quad \text{(V)}.
\]

Problem 6.12 The electromagnetic generator shown in Fig. 6-12 is connected to an electric bulb with a resistance of 100 \( \Omega \). If the loop area is 0.1 m\(^2\) and it rotates at 3,600 revolutions per minute in a uniform magnetic flux density \( B_0 = 0.2 \) T, determine the amplitude of the current generated in the light bulb.

Solution: From Eq. (6.38), the sinusoidal voltage generated by the a-c generator is

\[
V_{\text{emf}} = A \omega B_0 \sin(\omega t + C_0) = V_0 \sin(\omega t + C_0). \quad \text{Hence,}
\]

\[
V_0 = A \omega B_0 = 0.1 \times \frac{2\pi \times 3,600}{60} \times 0.2 = 7.54 \quad \text{(V)},
\]

\[
I = \frac{V_0}{R} = \frac{7.54}{100} = 75.4 \quad \text{(mA)}.
\]

Problem 6.13 The circular disk shown in Fig. 6-24 (P6.13) lies in the x-y plane and rotates with uniform angular velocity \( \omega \) about the z-axis. The disk is of radius \( a \) and is present in a uniform magnetic flux density \( B = 2B_0 \). Obtain an expression for the emf induced at the rim relative to the center of the disk.