1. Dr. Watson claims that he can light his entire kitchen with just one lamp if and only if the lamp is placed somewhere on a square table in the center of the kitchen. Draw a possible picture of his kitchen’s shape.

2. Sherlock Holmes claim that in his kitchen, there are to square tables, not touching each other; and he can light his whole kitchen with only one lamp if and only if the lamp is placed on just one of the tables, and not the other. Is this possible?

3. Place natural numbers on the vertices of a cube in such a way that the numbers on neighboring vertices (vertices connected by an edge) differ by no more than 1. Prove that there must be two diametrically opposite vertices whose numbers differ by no more than 1.

4. A greedy pirate keeps his gold coins in 77 treasure chests. Once, while he was counting his money, he noticed that if he opens any two chests, he can split the coins equally between them. Then he noticed that he can do the same with any 3, any 4, any 5, all the way up to any 76 chests. At this point, there was a knock on the door, and the old miser had no chance to check whether he could do the same for all 77 chests. Is it possible to determine this without looking at the chests?