Question:

Find real constant $A$ and complex constants $B$ and $C$ such that

$$\frac{10x^2 - 2x + 4}{x^2 + 1} = \frac{A}{x} + \frac{B}{x - i} + \frac{C}{x + i}$$

Solution:

Factoring the denominator, we have

$$\frac{10x^2 - 2x + 4}{x(x^2 + 1)} = \frac{10x^2 - 2x + 4}{x(x - i)(x + i)} = \frac{A}{x} + \frac{B}{x - i} + \frac{C}{x + i}$$

Equating the denominators on the right hand side, we get

$$\frac{10x^2 - 2x + 4}{x(x - i)(x + i)} = \frac{A(x - i)(x + i) + Bx(x + i) + Cx(x - i)}{x(x - i)(x + i)}$$

Reorganizing the numerator gives

$$\frac{10x^2 - 2x + 4}{x(x - i)(x + i)} = \frac{(A + B + C)x^2 + i(B - C)x + A}{x(x - i)(x + i)}$$

Equating the coefficients of the like terms on both sides, we end up with the following three equations

$$A + B + C = 10$$
$$i(B - C) = -2$$
$$A = 4$$

Replacing $A$ in the first equation and arranging it, we have two equations with two unknowns, $B$ and $C$

$$B + C = 6$$
$$i(B - C) = -2$$

Finally, solving these equations we obtain

$$B = 3 + i$$ and $$C = 3 - i$$

Congratulations!!!

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