Formal Languages and Automata Theory
Homework # 3

For each of the following, give a DFA that accepts the specified language. Exercises to be handed in from Part I include 1,3 and 28. Exercises to be handed in from Part II include 4 and 6.

Part I

1. The set of strings over \{a, b, c\} in which all a’s preceded the b’s, which in turn precede the c’s.
2. The same as in 1 except for the null string.
3. The set of strings over \{a, b\} in which the substring aa occurs exactly once.
4. The set of strings over \{a, b\} that do not contain the substring aaaa.
5. The set of strings over \{a, b, c\} that do not contain the substring aa.
6. The set of strings over \{a, b, c\} that begin with an a, contain exactly two b’s, and end with cc.
7. The set of strings over \{a, b, c\} in which the total number of b’s and c’s is three.
8. The set of strings over \{a, b, c\} in which every b is followed by at least one c.
9. The set of strings over \{a, b\} that contain the substring aa and the substring bb.
10. The set of strings over \{a, b, c\} that contain the substrings aa, bb, and cc.
11. The set of strings over \{a, b, c\} with length three.
12. The set of strings over \{a, b, c\} with length less than three.
13. The set of strings over \{a, b, c\} with length greater than three.
14. The set of strings over \{a, b\} in which the number of a’s is divisible by three.
15. The set of strings over \{a, b\} in which every a is either preceded or followed by a b, for example, baab, aba, and b.
16. The set of strings over \{a, b\} with an even number of a’s or an odd number of b’s.
17. The set of strings over \{a, b\} with an even number of a’s and an even number of b’s.
18. The set of strings over \{a, b\} that have odd length and contain exactly two b’s.
19. The set of strings over \{a, b, c\} that have odd length and contain exactly one a.
20. The set of strings over \{a, b, c\} with an odd number of occurrences of the substring ab.
21. The set of strings over \( \{a, b\} \) ending with the substring \textit{abba}.

22. The set of strings over \( \{1, 2, 3\} \) the sum of whose elements is divisible by 6.

23. The set of strings over \( \{a, b, c\} \) in which the number of a’s plus the number of b’s plus twice the number of c’s is divisible by six.

24. The set of strings over \( \{a, b\} \) in which every substring of length four has exactly one b.

25. The set of strings over \( \{a, b\} \) that contain an even number of substrings \textit{ba}.

26. The set of strings over \( \{0, 1\} \) in which every consecutive sequence of 3 symbols contains at least two zeros.

Part II

For problems 1-6, give an NFA that accepts the specified language. Note that in many cases it may be possible to simply give a DFA, which by definition is an NFA. However, in all cases you should try to exploit non-determinism in the construction of your NFA. For an extra exercise you might also try and come up with a DFA as well.

1. The set of strings over \( \{a, b\} \) whose third to the last symbol is \textit{b}.

2. The set of strings over \( \{a, b\} \) that have both or neither \textit{aa} and \textit{bb} as substrings.

3. The set of strings over \( \{a, b\} \) in which the substring \textit{aa} occurs at least once.

4. The set of strings over \( \{a, b\} \) in which the substring \textit{aa} occurs at least twice.

5. The set of strings over \( \{a, b\} \) ending with the substring \textit{abba}.

6. The set of strings over \( \{a, b\} \) containing an even number of occurrences of the substring \textit{ba}.