Modification of the lower ionospheric conductivity by thunderstorm electrostatic fields

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Abstract This paper reports a modeling study of the modifications of the nighttime lower ionospheric conductivity by electrostatic fields produced by underlying thunderstorms. The model used combines Ohm’s law with a simplified lower ionospheric ion chemistry model to self-consistently calculate the steady state nighttime conductivity above a thunderstorm. The results indicate that although the electron density is generally increased, the lower ionospheric conductivity can be reduced by up to 1–2 orders of magnitude because electron mobility is significantly reduced due to the electron heating effect. For a typical ionospheric density profile, the resulting changes in the reflection heights of extremely low frequency and very low frequency waves are 5 and 2 km, respectively.

1. Introduction

The conductivity of the lower ionosphere plays an important role in the propagation of extremely low frequency (ELF, 0.003–3 kHz) and very low frequency (VLF, 3–30 kHz) waves in the Earth-ionosphere waveguide. The fact that these waves are reflected in the D region of the ionosphere has been utilized to measure the electrical properties of this region. Recent remote sensing studies [e.g., Han and Cummer, 2010; Shao et al., 2012; Lay et al., 2014] have found that thunderstorms can modify the lower ionosphere on a time scale on the order of minutes to hours. Han and Cummer [2010] used broadband VLF signals (called sferics) excited by distant lightning discharges to probe the nighttime D region ionosphere along the wave propagation path to a sensor. The ionospheric height was obtained by fitting a series of simulated sferic spectra to the measured sferic spectrum. They found that the hourly average height of the ionosphere varies between 82.0 and 87.2 km over two summer months. Their measurements also indicate that the nightly variation of the ionosphere height averaged over 5 min is dominated by a decreasing pattern. The correlation between the variation of the ionosphere height with the lightning activity under the probed region on some nights indicates that lightning and thunderstorms can directly affect the lower ionosphere [Han and Cummer, 2010]. Shao et al. [2012] and Lay et al. [2014] compared the observed VLF/LF time waveforms of lightning strokes to the corresponding VLF/LF simulations to probe the nighttime D region that was situated approximately midway between the lightning discharges and receivers. For the study presented in Shao et al. [2012], the probed region was above a small thunderstorm. Over its lifetime, the electron density of the overhead D region below ~82 km altitude was concluded to be reduced by several orders of magnitude, and the extent of the reduction was closely related to the rate of lightning discharges in the storm [Shao et al., 2012]. On the other hand, Lay et al. [2014] probed the nighttime D region electron density profiles near and atop a large mesoscale thunderstorm over a 6 h period. They found that compared to the quiet nighttime condition, the lower ionosphere above the thunderstorm was generally less steep and its height was increased by 1.6–3.3 km. Their analysis indicated that these changes were not directly correlated with the rate of lightning within 200 km below the probing region but positive correlation was found for the probing region directly atop a lightning active area of the storm.

Thunderstorms are known to produce electric field and current in the overhead region that may penetrate into the lower ionosphere and cause significant modification. Although the correlation between the conduction current (i.e., the Wilson conduction current) above a thunderstorm and the lightning rate is still under debate [e.g., Mach et al., 2009, 2010; Mallios, 2015], the measured field and the total current (i.e., the Maxwell current that is the sum of the conduction and displacement current) are strongly correlated with the lightning activity [Blakeslee et al., 1989]. Because the displacement current vanishes at D region altitudes due to small local...
Maxwellian relaxation time (tens of ms at 70 km altitude [Liu et al., 2015]), the total current is dominated by the conduction component and therefore the lightning-correlated variations in the above mentioned studies may be related to the effects of thunderstorm electrostatic fields.

We recently reported a modeling study of the effects of thunderstorm electrostatic fields on the steady state ionospheric density of the D region [Salem et al., 2015]. The modeling results from a simplified ion chemistry model indicate that the steady state nighttime electron density can be enhanced by a factor up to ~6 whereas the maximum reduction is ~40%. However, this study did not provide a self-consistent solution to the electric field and electron density above a thunderstorm. In addition, to directly compare with the results obtained by using the VLF/LF probing technique, the modification of the D region conductivity needs to be examined.

Pasko et al. [1998] developed a 2-D cylindrically symmetric numerical model to self-consistently calculate the effect of the electron heating due to the thunderstorm electrostatic field on the ionospheric conductivity. The conductivity was found to decrease by up to 1 order of magnitude at altitudes from ~70 km to ~80–85 km, depending on the ambient conductivity profile. More recently, Kabirzadeh et al. [2015] have developed a 3-D numerical model to determine the effects of the geomagnetic field dip angle (the latitudinal dependence) on the mapping of the thunderstorm electrostatic fields to high altitudes and on the ionospheric conductivity due to electron heating. The results show stronger electric fields at high altitudes, particularly above 85 km, for lower geomagnetic latitudes. These stronger fields lead to more reduction in the self-consistently calculated ionospheric conductivity compared to the region at higher geomagnetic latitudes. It should be noted that these studies did not consider the chemical kinetics of the ionospheric ions in calculating the conductivity, and therefore the changes in the ionospheric density due to the modified ion chemical kinetics, found by Salem et al. [2015], were not taken into account.

In this paper, we develop a new model in order to self-consistently calculate the steady state conductivity of the lower ionosphere above a thunderstorm. Along with the effect of the three-body electron attachment process considered in our previous study [Salem et al., 2015], the new model takes into account the heating effects of thunderstorm electrostatic fields on the ionospheric electrons. The modeling results indicate that the lower ionospheric electron density can be increased by a factor up to 4–5 due to the thunderstorm electrostatic field, because the three-body attachment slows down. In contrast, the lower ionospheric conductivity is reduced by up to 1–2 orders of magnitude because of the decreased electron mobility. As a result, for a typical ionospheric density profile, the nighttime ionospheric reflection heights of ELF and VLF waves are 5 and 2 km, respectively.

2. One-Dimensional Steady State Conductivity Model

Our study is conducted by using the same simplified ion chemistry model as Salem et al. [2015], which was developed by Liu [2012, 2014]. In addition, a 1-D model is used to evaluate the steady state conductivity of the lower ionosphere above a thunderstorm that is modeled as a current source. Assuming a planar geometry, the total (Maxwell) current in the lower ionosphere is dominated by the conduction current and is equal to the total current at thunderstorm altitudes that is the difference between the charging current due to convection and the average discharging current due to lightning.

The steady state conductivity of the lower ionosphere can be self-consistently calculated by solving the following equations:

\[ E = \frac{J}{\sigma}, \]  
\[ \frac{dn_i}{dt} = 0 = S_i - L_i, \]  
\[ \sum_i n_i^+ = \sum_i n_i^-, \]  
\[ \sigma = \sum_i e n_i \mu_i. \]
where $E$ is the magnitude of the electric field established at a specific altitude above the thunderstorm, $\sigma$ the ionospheric conductivity at that altitude, and $J$ the current density; $n_i$ the density of the $i$th charged species, and $S_i$ and $L_i$ the source and loss for the $i$th species, respectively; $n_i^+$ the density of the positively charged species and $n_i^-$ the density of the negatively charged species; $e$ the electron charge and $\mu_i$ the mobility of the $i$th charged species.

$S_i$ and $L_i$ are formulated by using the ion chemistry model, as discussed in section 2 [Salem et al., 2015], with a given ionization source profile $Q_i$ and $J$ is externally specified. The system of equations represented by equation (2) is not independent, and one of them is replaced by the charge neutrality condition (equation (3)). The nonlinear equation system represented by equations (1)–(4) can be solved numerically by using a numerical solver such as the KINSOL module of the SUNDIALS software package developed by Lawrence Livermore National Laboratory (http://computation.llnl.gov/casc/sundials/main.html).

3. Field-Dependent Quantities: Three-Body Electron Attachment and Electron Mobility

Equation (4) indicates that the conductivity is proportional to the densities and mobilities of the charged species, which can both change due to the thunderstorm electrostatic field. In our ion chemistry model, the rate constant ($k_{3ab}$) of the three-body electron attachment is dependent on the average electron energy ($\bar{\epsilon}$) or the reduced electric field ($E/N$) [Salem et al., 2015], as shown in Figure 1a. As a result, the nighttime electron density can be reduced by up to ~40% or enhanced by a factor up to ~6, when the field varies from 0 to 0.4$E_T$ [Salem et al., 2015]. It should be noted that the calculation was made by specifying the electric field at each altitude and therefore did not provide a self-consistent density profile for a given thunderstorm charging current. The thunderstorm electrostatic field can also significantly modify the electron mobility, which is discussed below. For ions, the change in their mobilities is negligible because of their large mass [Davies, 1983].

A comparison of the electron mobility ($\mu_e$) as a function of $E/N$ obtained or used by various experimental and theoretical studies [e.g., Pack et al., 1962; Baum, 1965; Wyatt, 1967; Rees, 1973; Dutton, 1975; Hegerberg and Reid, 1980; Davies, 1983; Roznerski and Leja 1984; Lisovskiy and Yegorenkov, 1998; Pasko et al., 1997] is shown in Figure 1b. In addition, $\mu_e$ calculated from BOLSIG+ and air1 are also shown in the same figure. BOLSIG+ is a useful tool obtained by solving the Boltzmann equation for electrons in weakly ionized gases in uniform electric fields [Hageleaar and Pitchford, 2005] and can be freely downloaded at http://www.bolsig.laplace.univ-tlse.fr. MATLAB function air1 results from the work of Moss et al. [2006], which can be used to find the coefficients for the most important electron impact processes in air, and it is available at http://pasko.ee.psu.edu/air/. It can be seen that $\mu_e$ from different sources is, to a large extent, consistent above ~5 Td (Townsend). For the purpose of our study, we use the following functional dependence that fits well with the experimental data reported:

$$
\mu_e = N_0 \mu_{eo} \sum_{i=0}^{3} \frac{a_i x^i}{\beta_i x^i},
$$

where $x = E/N$ (the reduced electric field) in Townsend, $E$ is the electric field, $N$ the atmospheric neutral density (MSIS profile, http://omniweb.gsfc.nasa.gov/vitmo/msis_vitmo.html), $a_0 = 1$, $a_1 = 9.43 \text{Td}^{-1}$, $a_2 = 27.94 \text{Td}^{-2}$, $a_3 = 1.46 \text{Td}^{-3}$, $\beta_0 = 1$, $\beta_1 = 11.41 \text{Td}^{-1}$, $\beta_2 = 176.22 \text{Td}^{-2}$, $\beta_3 = 77.46 \text{Td}^{-3}$, the zero-field mobility at sea level $\mu_{eo} = 1.75 \text{m}^2\text{V}^{-1}\text{s}^{-1}$ [Milloy et al., 1975], and the atmospheric density at sea level $N_0 = 2.688 \times 10^{23} \text{m}^{-3}$.

4. Modeling Results

We consider the same nighttime ionization source profiles ($Q_1$ and $Q_2$) presented in the work of Salem et al. [2015], but due to limited space, we only discuss the results for $Q_1$ below (the results for $Q_2$ that is derived from measured cosmic ray ionization rate profile [Brasseur and Solomon, 2005, p. 553] and that produces steeper electron density profile are available as supporting information). Figure 2 shows the self-consistent ionospheric altitude profiles of the reduced electric field $E/N$, the electron density $n_e$, and the total conductivity $\sigma$ at various $J$ values up to $10^{-8} \text{A/m}^2$ that represents the maximum total current density in thunderstorms [Riousset et al., 2010].
Figure 1. (a) The rate constant of three-body electron attachment $k_{3\text{att}}$ as a function of average electron energy or $E/N$ [Salem et al., 2015]. The light (dark) gray area represents the region where the electron density is expected to be reduced (increased), compared to its background profile, due to the variation of $k_{3\text{att}}$. (b) The variation of electron mobility as a function of $E/N$ from the experimental and theoretical studies available in the literature [e.g., Pack et al., 1962; Baum, 1965; Wyatt, 1967; Rees, 1973; Dutton, 1975; Hegerberg and Reid, 1980; Davies, 1983; Roznerski and Leja, 1984; Lisovskiy and Yegorenkov, 1998; Pasko et al., 1997]. The fitting formula (equation (5)) is shown by thick black line.

Although $E$ (not shown in the figure) decreases with increasing altitude, $E/N$ does not necessarily follow the same variation. In general, when $J > 10^{-11}$ A/m², $E/N$ increases with increasing altitude up to ~64 km above which it begins to decrease rapidly with some local peaks between 72 and 76 km. This altitude dependence can be explained as follows. From equation (1),

$$\frac{E}{N} = \frac{J}{\sigma N} \sim \exp \left(-\frac{z}{H_e} + \frac{z}{H_N}\right) \sim \exp \left(-\frac{z(H_e - H_N)}{H_e H_N}\right).$$

Figure 2. The modeling results of the effects of the thunderstorm electrostatic field on the lower ionosphere when $J = 10^{-11}$, $10^{-10}$, $10^{-9}$, and $10^{-8}$ A/m². The altitude profiles of (a) reduced electric filed, (b) electron density, and (c) conductivity. The nighttime ionization source profile ($Q_1$) in Figure 2b is from Salem et al. [2015]. The ambient profiles (the dotted lines in Figures 2b and 2c) are obtained by setting $J = 0$. The ion conductivity profile ($\sigma_{\text{ion}}$) in Figure 2c is from measurements [Holzworth et al., 1985]. Below ~64 km altitude where the conductivity is dominated by ions, the modeling results agree well with $\sigma_{\text{ion}}$. Above 64 km altitude, the conductivity is dominated by the rapidly increasing electronic component, resulting in a large difference between $\sigma$ and $\sigma_{\text{ion}}$.
where \( H_e \) and \( H_N \) are the scale heights of increasing \( \sigma \) and decreasing \( N \), respectively. Below \( \sim 64 \) km altitude, \( \sigma \) is dominated by ions with \( H_e = 11 \) km (see Figure 2c) and \( H_N \) is roughly \( 7 \pm 1 \) km in the D region [Brasseur and Solomon, 2005, p. 64], which results in an increasing \( E/N \) profile. Above 64 km, the \( \sigma \) is dominated by electron component with \( H_e \) smaller than \( H_N \) and thus \( E/N \) decreases with altitude. When \( J \leq 10^{-11} \) A/m², \( E/N \) is too small to produce any noticeable effects.

Figure 2b shows the profiles of electron density. When \( J = 10^{-8} \) A/m², \( n_e \) increases by a factor of \( \sim 4-5 \) at the altitude up to \( \sim 75 \) km with the maximum increase occurring at 64 km altitude. Above 75 km, the change in \( n_e \) is small. For \( J = 10^{-9} \) A/m², \( n_e \) mainly increases, with a slight decrease near 75 km. On the other hand, when \( J = 10^{-10} \) A/m², \( n_e \) decreases by \( \sim 30-40\% \) at the altitude up to \( \sim 75 \) km with the maximum reduction occurring at 70 km altitude. There are no appreciable changes above 79 km altitude for all values of \( J \). The change in \( n_e \) at each altitude depends on the magnitude of the \( E/N \) value at that altitude because of the field-dependent \( K_{\text{sat}} \) [Salem et al., 2015]. As shown in Figure 1a, \( K_{\text{sat}} \) increases linearly with \( E/N \) field in the range of 0 to 0.1 Td while decreases exponentially from 0.1 Td to \( \sim 0.4E_k \). With this dependence, for \( E/N \) values in the range between 0 and 0.76 Td (the light gray area in Figures 1a and 2a), the corresponding \( K_{\text{sat}} \) values exceed their ambient value and the resulting \( n_e \) values in the range of 0 to 0.1 Td while decreases exponentially from 0.1 Td to \( \sim 0.4E_k \). This is the reason why for all values of \( J \) and \( E/N \) field, \( n_e \) is not reduced by a factor of \( \sim 4-5 \) at the altitude up to \( \sim 75 \) km. For \( J = 10^{-11} \) A/m², despite the enhancement in \( n_e \) at the altitude range of \( \sim 64-79 \) km, the conductivity is decreased by \( \sim 2 \) orders of magnitude around \( \sim 79 \) km altitude owing to the predominant reduction in \( \mu_e \). It should be noted that the change in \( \sigma \) brings \( H_N \), between \( \sim 72 \) and 76 km altitude, back to a value close to \( H_e \) below 64 km. This change results in the local peaks of the \( E/N \) field (Figure 2a), indicating better penetration of the thunderstorm electric field to higher altitudes. Consistently, \( \sigma \) is reduced even between 79 km and 85 km altitude, where change in \( n_e \) is negligible. The reduction in \( \sigma \) above 64 km altitude for significant \( J \) values is caused by the electron heating effect. As shown by Figure 1b, the electron mobility decreases quickly when electric field increases from 0 to 10 Td, which leads to the reduction in the conductivity. \( \sigma \) increases in the same way when \( J > 10^{-11} \) A/m², but it is reduced by a smaller factor and in a smaller altitude range because of the increased electron density. Note also that when \( J \leq 10^{-11} \) A/m², the corresponding \( E/N \) values are very small and the change in \( n_e \) is negligible. In addition, \( \sigma \) below \( \sim 64 \) km altitude remains equal to the ambient levels for all \( J \) values because it is dominated by ions, which are not affected by the electric field.

Many previous studies assume that the electron mobility or the electron neutral collision frequency \( (v_{ce} \sim \mu_e^{-1}) \) is independent from the electric field. As a comparison, Figure 3 shows the modeling results for the same cases discussed above, but with unvarying \( n_e \) with electric field \( (\mu_e = \mu_{e0}N_e/N_e, \text{see equation (5)}) \). Compared to the case of varying \( \mu_e \), \( E/N \) is reduced by up to \( \sim 1 \) order of magnitude at 64 km altitude and above, when \( J \leq 10^{-8} \) A/m². In addition, there are no local peaks showing better penetration of the electric field to higher altitudes. The changes in \( \sigma \), compared to the case of varying \( \mu_e \) are smaller. With constant \( \mu_e \), the change in \( \sigma \) is directly proportional to the change in \( n_e \). The slightly increased \( \sigma \) results in the evident reduction in the \( E/N \) values above 64 km, compared to Figure 2a.

5. Discussion

5.1. Reflection Height of ELF/VLF Waves

The ELF/VLF waves propagating in the Earth-ionosphere waveguide are reflected by the D region of the ionosphere. According to our modeling results shown in Figure 2, the thunderstorm field can significantly modify the conductivity of this region, and may therefore change the reflection height of the ELF/VLF waves. For a given conductivity profile, the refractive index \( n \) of the lower ionosphere, in the absence of the geomagnetic field, can be approximated by [e.g., Ratcliffe, 1959; Kenneth, 1966; Marshall, 2009]

\[
n^2 \simeq 1 - \frac{\omega_p^2}{\omega_{ce}^2},
\]

where \( \omega_p \) is the plasma frequency, \( \omega \) the angular wave frequency, and \( \nu_{ce} \) the electron neutral collisions frequency. The plasma frequency is defined by \( \omega_p^2 = e^2n_e/\epsilon_0m_e \), where \( \epsilon_0 \) is the permittivity of free space.
Figure 3. The modeling results of the effects of the thunderstorm electrostatic field on the lower ionosphere when $J = 10^{-11}, 10^{-10}, 10^{-9},$ and $10^{-8}$ A/m$^2$ and $\mu_e$ is assumed to be constant. Altitude profiles of (a) reduced electric field, (b) electron density, and (c) conductivity.

and $m_e$ the electron mass. The electron neutral collisions frequency $\nu_e = e/m_e \mu_e$. Ratcliffe [1959, p. 90] shows that the reflection of ELF/VLF waves in the lower ionosphere occurs at the altitude where the real and imaginary parts of $n^2$ are equal; i.e., $\omega \equiv \omega_e^2/\nu_e$. The nighttime ionospheric reflection heights, determined by using equation (6), and the conductivity profile from $J=10^{-8}$ A/m$^2$ for a wide range of frequencies, including ELF and VLF bands, are shown in Figure 4. For further comparison, we consider both cases of varying $\mu_e$ and constant $\mu_e$, which are shown in Figures 4a and 4b, respectively. The change of the reflection height ($\Delta H$) at each frequency is shown in Figures 4c and 4d. For the ambient nighttime ionosphere (the dotted line in Figures 4a and 4b), ELF waves are reflected at altitudes between $\sim 68$ and $\sim 80$ km, whereas VLF waves are reflected at altitudes between $\sim 80$ and $\sim 84$ km. Figures 4a and 4c show that the ELF and VLF reflection heights

Figure 4. The nighttime ionospheric reference height for $J = 0$ and $10^{-8}$ A/m$^2$: (a) varying $\mu_e$ with the field and (b) constant $\mu_e$. The change in the nighttime ionospheric reference height ($\Delta H$): (c) varying $\mu_e$ with the field and (d) constant $\mu_e$. The sign of $\Delta H$ is positive (negative) if the reflection height is elevated (lowered) compared to the ambient reflection height.
are elevated by up to ~5 and ~2 km, respectively, compared to their ambient reflection heights. In contrast, only the reflection heights of ELF waves are changed with a reduction up to ~2 km, as shown in Figures 4b and 4d. This indicates that the constant mobility assumption can lead to significant errors when investigating the effects of the thunderstorm electric field on the ionosphere conductivity. The discussion above applies to the ionospheric density profile that is often used to measure the changes in the height of the lower ionosphere under the influence of thunderstorm lightning activities [e.g., Han and Cummer, 2010; Lay et al., 2014]. For a steeper ionospheric density profile like the one discussed in the supporting information, the modified profile is even steeper, and the changes of the reflection heights are smaller but the conductivity is still reduced by up to 1 order of magnitude at altitudes around ~75 km. On the other hand, Shao et al. [2012] used an ambient density profile with a steepness between those two cases. Although the reflection heights for VLF waves are only increased by up to ~0.5 km, the conductivity is reduced by up to 2 orders of magnitude at altitudes around ~77 km, which is consistent with their measurements.

Finally, it should be noted that the effects of the geomagnetic field (B) are neglected in our calculation. This is reasonable when the term $\mu_e B^2$ is much less than 1; i.e., the electron neutral collision frequency is much higher than the electron gyrofrequency [e.g., Pasko et al., 1998; Kabirzadeh et al., 2015]. For quiet ionospheric condition, $\mu_e B^2$ is less than 1 below ~70 km altitude, and for $J = 10^{-6}$ A/m$^2$, this altitude is increased to ~82 km due to decreased mobility. According to Kabirzadeh et al. [2015], electric field increases by a factor of ~6–40 in the altitude range of 80–90 km, when the magnetic field effects are included. We use this factor to scale the electric field obtained by our model to estimate the additional reduction in the electron mobility because of better penetration of the electric field enabled by the magnetic field effects. For $J = 10^{-6}$ A/m$^2$, the conductivity in the aforementioned altitude range is further reduced by a factor of about 0.3. As a result, the reflection height of VLF waves is increased by an additional 0.5 km but no changes to the reflection height of ELF waves. In addition, no appreciable changes in the ELF/VLF wave reflection height are found when the calculations are repeated for the case discussed in the supporting information.

5.2. Comparison With Observations

This study indicates that the electrostatic fields produced by thunderstorms are significant for electrical properties of the lower ionosphere up to ~85 km altitude. Under steady state conditions, our modeling results indicate that the nighttime lower ionospheric conductivity can be reduced by up to 1–2 orders of magnitude, depending on the nighttime ionospheric electron density profile, due to thunderstorm electrostatic fields.

We conclude that this change in the conductivity is attributed to electron heating effect because the lower ionospheric electron density is generally increased under the influence of those fields (see also Salem et al. [2015]). The reduction in the conductivity is in a good agreement with both theoretical results reported in Pasko et al. [1998], and the experimental findings of Shao et al. [2012], assuming what is measured by using the VLF/LF probing technique is the ionospheric conductivity rather than the electron density. Moreover, our study suggests that under the effects of the thunderstorm electrostatic field, the changes in the measured ionospheric height do not reflect the changes in the electron density. For instance, as a result of the reduction in the lower ionospheric conductivity, the nighttime ionospheric height measured by using the VLF waves reflection technique can be increased by ~2 km despite the increase in the electron density.

Given the above discussion, the observed decrease in the ionosphere height due to an underlying thunderstorm reported by Han and Cummer [2010] and Lay et al. [2014] could be due to a significant enhancement in the electron density, which may be caused by transient luminous events triggered by intense lightning discharges.

References

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