Consider a cascaded nonlinear stage.

One way write:

\[ y_{1}(t) = a_{11} x(t) + a_{21} x^2(t) + a_{31} x^3(t) \]  \( (* *) \)

\[ y_{2}(t) = a_{12} y_1(t) + a_{22} y_1^2(t) + a_{32} y_1^3(t) \]  \( (*) \)

where \( a_{ij} \) denotes the \( i \)th order gain of the \( j \)th stage.

After substitution, \( (* *) \) into \( (*) \), one obtains:

\[ y_{2}(t) = a_{12} \left( a_{11} x(t) + a_{21} x^2(t) + a_{31} x^3(t) \right) + \]
\[ a_{22} \left( a_{11} x^2(t) + a_{21} x^3(t) \right)^2 + \]
\[ a_{32} \left( a_{11} x^3(t) + a_{21} x^2(t) + a_{31} x^3(t) \right)^3 \]

To find third order intercept point, one needs to find the linear and the third order term. After some manipulation, one obtains:

\[ y_{3}(t) = A_1 x(t) + A_2 x^2(t) + A_3 x^3(t) \]

where \( A_1 = a_{11} a_{12} \)
\[ A_2 = a_{21} a_{22} + 2a_{11} a_{21} a_{22} + a_{11}^3 a_{22} \]
For a single stage, the third order intercept point is given by

\[
A_{IP3} = \sqrt{\frac{A}{3 - \left| \frac{a_{11} a_{12}}{a_{31} a_{12} + 2 a_{11} a_{21} a_{22} + a_{11} a_{32}} \right|}}
\]

Therefore, by applying the same principle to the cascaded system, one obtains

\[
A_{IP3C} = \left[ \frac{A}{3 - \left| \frac{a_{11} a_{12}}{a_{31} a_{12} + 2 a_{11} a_{21} a_{22} + a_{11} a_{32}} \right|} \right]^{1/2}
\]

The signs of the terms in the denominator are case by case dependent. The worst case scenario occurs when all the terms are of the same sign. For such cases, \( A_{IP3C} \) is the smallest. Considering the worst case, one can rewrite (*) as

\[
\frac{1}{A_{IP3C}^2} = \frac{3}{4} \left| \frac{a_{31} a_{12}}{a_{11} a_{12}} \right| + \frac{3}{2} \left| \frac{a_{11} a_{22}}{a_{11} a_{12}} \right| + \frac{3}{4} \frac{a_{11}^2}{a_{11} a_{12}}
\]

One observes that

\[
\frac{3}{4} \left| \frac{a_{31} a_{12}}{a_{11} a_{12}} \right| = \frac{3}{4} a_{11}^2 \quad \frac{3}{2} \left| \frac{a_{11} a_{22}}{a_{11} a_{12}} \right| = \frac{3}{2} a_{12}^2 \quad \frac{3}{4} \frac{a_{11}^2}{a_{11} a_{12}} = \frac{a_{11}^2}{A_{IP3,1} A_{IP3,2}}
\]

Therefore,

\[
\frac{1}{A_{IP3C}^2} = \frac{1}{A_{IP3,1}} + \frac{a_{11}^2}{A_{IP3,2}} + \frac{3}{2} \left| \frac{a_{11} a_{22}}{a_{11} a_{12}} \right| \quad (\star)
\]

In most systems, \( a_2 \) terms are small, and \( a_1 \) terms are large. (This is usually by design and due to symmetry requirements.) Therefore, in most practical situations the last term of (\star) may be neglected, and the expression becomes

\[
\frac{1}{A_{IP3C}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{a_{11}^2}{A_{IP3,2}^2}
\]
Useful observation: If \( a_{1}^{2} \) is large (gain of the first stage), the IP3 of the cascaded system depends mostly on the output stage of the cascaded system.

The equation (XXX) may be extended to multiple nonlinear systems yielding approximate relationship for overall IP3 of the system:

\[
\frac{1}{A_{1P3_c}} = \frac{1}{A_{1P3,1}} + \frac{a_{1}^{2}}{A_{1P3,2}} + \frac{a_{1}^{2}a_{2}^{2}}{A_{1P3,3}} + \cdots + \frac{a_{1}^{2}a_{2}^{2}\cdots a_{n-1}^{2}}{A_{1P3,n}} \tag{X}
\]

Keeping in mind that the input power at the IP3 may be expressed as:

\[
II_{P3} = \frac{A_{1P3}}{R} \quad (R \text{ usually taken as } 50\, \Omega)
\]

One way rewrite (X) as:

\[
\frac{1}{II_{P3_c}} = \frac{1}{II_{P3,1}} + \frac{G_{1}}{II_{P3,2}} + \frac{G_{1}G_{2}}{II_{P3,3}} + \cdots + \frac{G_{1}G_{2}\cdots G_{n-1}}{II_{P3,n}} \tag{XX}
\]

Note: The overall intercept point is dominated by the intercept point of the output stage.

Sometimes it is useful to express the third-order intercept point as the function of the output power.

For each individual stage one way write:

\[
OP_{3,c} = a_{i}^{2} \cdot II_{P3,c} \Rightarrow II_{P3,c} = OP_{3,i} / a_{i}^{2}
\]

By substitution into (XX) one obtains:

\[
\frac{G_{1}G_{2}\cdots G_{n}}{OP_{3,c}} = \frac{1}{OP_{3,1}} + \frac{G_{1}}{OP_{3,2}/G_{1}} + \frac{G_{1}G_{2}}{OP_{3,3}/G_{2}} + \cdots + \frac{G_{1}G_{2}\cdots G_{n-1}}{II_{P3,n}/G_{n}}
\]
\[
\frac{1}{OIP_3} = \frac{1}{G_0} + \frac{1}{G_n OIP_{n+1}} + \frac{1}{G_b G_n OIP_{n+2}} + \ldots + \frac{1}{OIP_{n+1}}
\]

If the gains are large, one obtains \( OIP_3 \approx OIP_{n+1} \).

**Gain compression (1dB compression point)**

Consider a conceptual block diagram of any active two port network:

\[
\text{Source} \xrightarrow{\text{Pin}} \text{Active device} \xrightarrow{\text{Pout}} \text{Load} \xrightarrow{\text{dissipative Ploss}}
\]

\[ \text{Pin} + \text{Pdc} = \text{Pout} + \text{Ploss} \]

on the other hand

\[ \text{Pout} = G_p \cdot \text{Pin} \quad G_p = \text{power gain} \]

Therefore

\[ G = 1 + \frac{\text{Pdc} - \text{Ploss}}{\text{Pin}} \]

Since \( \text{Pdc} \) is bounded by capabilities of power supply as \( \text{Pin} \) increases the gain has to decrease at some point. As a result all active devices must be nonlinear.

Consider now the output of general nonlinearity when presented with a single tone:

\[ x(t) = X \cos(\omega t) \rightarrow s(x) = a_1 x + a_2 x^2 + \ldots \rightarrow y(t) = \left( a_1 A + \frac{3a_3 A^2}{2} \right) \cos(\omega t) + \ldots \]
In worst cases \( a_3 \) is negative and it is easy to see that as \( A \) increases, the output amplitude becomes smaller relative to ideal, amplified form \( a_1 A \).

To exactly the nonlinearity we use 1dB compression point.

At the 1dB compression point one has:

\[
\left( A_1 - \frac{3a_3 A_1^2}{4} \right) \left( \frac{3a_3 A_1^2}{4} \right) = a_1 - 1\text{dB}
\]

\[
2 \log (A_1 + \frac{3a_3 A_1^2}{4}) = 2 \log (1a_1) - 2 \log (1.122)
\]

\[
1a_1 - \frac{3a_3 A_1^2}{4} = \frac{1a_1}{1.122}
\]

Solving for \( A_1 \) one obtains:

\[
A_1 = \sqrt{0.145} \left| \frac{a_1}{a_3} \right|
\]

Comparing the expressions for \( A_4 \text{dB} \) and \( A_{1\text{dB}} \) one has:

\[
A_1 \left( \sqrt{0.145} \left| \frac{a_1}{a_3} \right| \right) = \sqrt{\frac{3 \cdot 0.145}{4}}
\]

or

\[
\frac{A_1}{A_{1\text{dB}}} = 0.
\]

or

\[
P_1 [\text{dBm}] = 11P_3 [\text{dBm}] - 9.7 \text{ dB}
\]

As a practical rule one usually assumes that \( P_1 [\text{dBm}] \) is 10 dB below the third order intercept point for a single tone test.
Example: Consider the receiver block diagram given below. Estimate the overall 1IP3.

\[ \text{L} = 2 \text{dB} \quad G = 10 \text{dB} \quad \text{L} = 5 \text{dB} \quad G = 20 \text{dB} \]

1IP3 = \infty \quad 1IP3 = 12 \text{dBm} \quad 1IP3 = 14 \text{dBm} \quad 1IP3 = 16 \text{dBm}

Steps:
1. Convert all gains, losses, and 1IP3 to 
   linear domain.

BP A. L = 2 dB, G = 2 dB, \( q_1 = 0.63 \), 1IP3 = \infty

LNA:
\[ G = 10 \text{dB}, \quad q_2 = 10 \quad 1IP3_2 = 10^{0.63 \times 10} = 15.84 \text{ mW} \]

MixeR:
\[ L = 5 \text{dB} \quad G = -5 \text{dB} \quad q_3 = 0.8162 \quad 1IP3_3 = 10^{0.63 \times 10 \times 0.8162} = 25.12 \text{ mW} \]

IF A. M.:
\[ G = 20 \text{dB} \quad q_3 = 100 \quad 1IP3_4 = 10^{0.63 \times 10} = 39.81 \text{ mW} \]

\[ \frac{1}{1IP3} = 0.63 + \frac{0.63}{15.84 \text{ mW}} + \frac{0.63 \times 10}{25.12 \text{ mW}} + \frac{0.63 \times 10 \times 0.8162}{39.81 \text{ mW}} \]

\[ \frac{1}{0.3406 \text{ (mW)}^{-1}} \Rightarrow 1IP3 = 2.9391 \text{ mW} \rightarrow 46.7 \text{ dBm} \]
Estimate of the 1 dB compression point

\[ P_{1\ dB_{\text{B}}^{-}} = 4.67 \text{ dB}_{\text{B}}^{-} - 10 \text{ dB} = -5.33 \text{ dB}_{\text{B}}^{-} \]

The signal at the input needs to be smaller than \(-5.33 \text{ dB}_{\text{B}}^{-}\).

**Blocking**

Occurs when a weak desired signal is mixed along with a strong interfering signal.

\[ x(t) = A_0 \cos(\omega t) + A \cos(\omega t + \phi) \]

\[ y(t) = A_1 x(t) + A_3 x^3(t) + \ldots \]

\[ \Rightarrow y(t) = \frac{A_0 + 3A_3 A_0^2 + 3A_3 A_1^2}{4} \cos(\omega t) + \frac{\phi}{\cos(\omega t)} \]

If \( A_1 \gg A_0 \), the output signal may be approximated as

\[ y(t) \approx \frac{A_0 + 3A_3 A_1^2}{2} \cos(\omega t) \]

Usually, \( A_0 \) is negligible and the amplitude gain is reduced by

\[ A_1 \rightarrow 19.1 - \frac{3.103}{1 \times 10^6} \]

Reduction of the system gain due to blocking is sometimes referred to as the RX desensitization. Most RX are required to withstand a level of adjacent channel interference. For example, in cellular systems the difference between the desired signal and adjacent interference may be \(-70 \text{ dB}\). Avoidance of blocking translates in high

\[ \text{signal-to-noise ratio (SNR)} \]