Small signal amplifiers (Review) - commonly used configurations

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BJT Circuits.

In the active region, the following equations hold:

\[ I_C = I_{CS} \exp \left( \frac{V_{BE}}{V_T} \right) (1 + \alpha V_{CE}) \]

\[ I_B = \frac{1}{\beta} I_C \]

always we have: \[ I_B + I_C = I_E \]

Small signal model (voltage and current changes around the operating point):

\[ r_J = \frac{\Delta V_{BE}}{\Delta I_B} = \frac{\partial (\frac{I_{ES} \exp (V_{BE})}{V_T})}{\partial V_{BE}} = \frac{\partial}{\partial V_{BE}} \left[ \frac{I_{CS} \exp (V_{BE})}{V_T} \right] = \frac{I_{CS} \exp (V_{BE})}{V_T} \]

\[ = \left( \frac{1}{\beta} - \frac{1}{V_T} \right) \frac{I_{CS} \exp (V_{BE})}{V_T} \]
\[
gm = \frac{\Delta I_c}{\Delta V_{BE}} = \frac{\partial I_c}{\partial V_{BE}} = \frac{2}{V_T} \left( I_C \cdot \exp \left( \frac{V_{BE}}{V_T} \right) \cdot (1 + \lambda V_{CE}) \right)
\]

\[
\frac{1}{V_T} \cdot I_C \cdot \exp \left( \frac{V_{BE}}{V_T} \right) \cdot (1 + \lambda V_{CE}) = \frac{1}{V_T} \cdot I_C
\]

\[
V_o = \frac{\Delta V_{CE}}{\Delta I_c} = \left( \frac{\partial I_c}{\partial V_{CE}} \right) = -\frac{2}{V_T} \cdot \left[ I_C \cdot \exp \left( \frac{V_{BE}}{V_T} \right) \cdot (1 + \lambda V_{CE}) \right]
\]

\[
= \left[ \frac{2}{V_T} \cdot I_C \cdot \exp \left( \frac{V_{BE}}{V_T} \right) \cdot (1 + \lambda V_{CE}) \right] = \frac{1}{\lambda V_C}
\]

Quantity \( \frac{1}{\lambda} \) is referred to as Early voltage. Therefore,

\[
V_o = \frac{V_A}{I_C}
\]

There other parameters of the small signal model that are omitted for now. They will be introduced later when high-frequency operation of the amplifier is considered.

**Common Emitter configuration**

![Diagram](attachment:diagram.png)

Emitter is connected to the AC ground.

We desire to determine:
1) Voltage gain.
2) Current gain.
3) Input impedance.
4) Output impedance.
Small signal model circuit: the voltage gain $V_a = \frac{V_o}{V_i}$.

From input circuit: $\Delta V_{be} = \frac{r_{n1}\|r_{n2}}{r_{n2} + r_{n1}\|r_{n2}} \Delta V_i$.

From output circuit: $V_o = -g_{w}\Delta V_{be} - R_o\|R_L$.

Substituting (1) into (2), one obtains:

$V_o = -g_{w} R_o\|R_L - \frac{r_{n1}\|r_{n2}}{r_{n2} + r_{n1}\|r_{n2}} R_s + r_{n1}\|r_{n2}$.

or

$A_v = \frac{V_o}{V_i} = -g_{w} \frac{R_L\|R_o}{R_o\|R_L + \frac{r_{n1}\|r_{n2}}{r_{n2} + r_{n1}\|r_{n2}}}$.

Simplification: Most commonly, $r_{n2} \gg R_L$.

Under these assumptions:

$A_v = -g_{w} R_L.$
2) $A_i = \frac{I_o}{I_i} - \text{current gain}$

From input circuit $V_{bc} = r_{st} \cdot I_e = \frac{R_{11}R_2}{R_{11}R_2 + r_{st}}$ (X)

From output circuit $I_o = -g_m V_{be} \cdot \frac{r_o}{R_{o+2r}}$ (X*).

Substituting (X*) into (X):

$I_o = -g_m \cdot \frac{r_o}{R_{o+2r}} \cdot \frac{r_{st} \cdot R_{11}R_2}{R_{11}R_2 + r_{st}} \cdot I_i$

Or

$A_i = \frac{I_o}{I_i} = -g_m \cdot \frac{r_o}{R_{o+2r}} \cdot \frac{r_{st} \cdot R_{11}R_2}{R_{11}R_2 + r_{st}} = -g_m \cdot \frac{r_o}{r_{st} + R_L} \cdot r_{st} \cdot R_{11}R_2$

Simplification: Most commonly $V_o \gg R_L$

Under these assumptions:

$A_i = -g_m \cdot r_{st} = -\beta$

3) Input impedance

$R_i = \frac{V_i}{I_i} = R_{11}R_2 + r_{st}$ from the schematic

4) Output impedance

$R_o = \frac{V_o}{I_o} = r_o R_L$

Therefore in the common emitter configuration, the amplifier may be represented...
with an equivalent circuit (other equivalent circuits are possible)

Example: Design a CE amplifier. Assume that the h不吃er operating point is $I_{c} = 1 \mu A$, $\beta = 100$. The Early voltage $V_A = 50 V$. Calculate voltage gain, current gain, input impedance, output impedance for a small signal input with $R_S = 50 \Omega$. The battery used for the design has $V_c = 12 V$

Biasing

Design guideline: About 10% of $V_c$ is used for stabilization. That is $V_E \approx 0.1 \cdot V_c$

$V_E = R_E \cdot I_{c} = 0.1 \cdot V_c \Rightarrow R_E = \frac{0.1 \cdot V_c}{I_{c}} = \frac{0.1 \cdot 12 V}{1 \mu A} = 12 k \Omega$

$I_B = I_c / \beta = 1 \mu A / 100 = 10 \mu A$

In the first approximation we will assume that $I_B \ll I_{c}$. Therefore it can be neglected and the base voltage is given by
\[ V_E = \frac{R_2}{R_2+R_1} \quad V_{cc} = V_E + V_{BE(on)} \]

\[ \frac{1}{1+\frac{R_1}{R_2}} = \frac{V_{E+V_{BE(on)}}}{V_{cc}} \quad \Rightarrow \quad \frac{R_1}{R_2} = \frac{V_{cc}}{V_{E+V_{BE(on)}}} - 1 \]

In this case \[ \frac{R_1}{R_2} = \frac{12\,V}{(12+0.68)\,V} = 5.48 \quad (4) \]

Let us assume \[ I_{BB} = 100\,I_B = 1\,\mu A \]

Therefore \[ R_1 + R_2 = \frac{12\,V}{1\,\mu A} = 12\,K \quad (2) \]

From (4) \[ R_1 = 5.48\,R_2 \]

From (2) \[ (5.48 + 1)R_2 = 12\,K \Rightarrow R_2 = \frac{12\,K}{6.48} = 1.85\,K \]

\[ R_1 = 5.48\cdot R_2 = 10.15\,K \]

Collector resistor is designed so that it accommodates largest possible amplitude swing.

\[ V_{c(cw)} = V_{cc} \]

\[ V_{c(m)} = V_E + V_{CE} + \varepsilon \quad \Rightarrow \quad R_2 = \frac{12\,K}{6.48} = 1.85\,K \]

\[ V_{c(cw)} = V_{cc} = 12\,V \]

\[ V_{c(m)} = \frac{1}{2}(12\,V + 0.3\,V + 0.7\,V) = 2.2\,V \]

\[ V_{cp} = \frac{V_{c(m)} + V_{c(cw)} - V_{c(m)}}{2} = \frac{2.2\,V + 12\,V - 2.2\,V}{2} = 6.9\,V \]

\[ V_{ce} = V_{cc} - I_{ce}R_c \Rightarrow R_c = \frac{V_{cc} - V_{ce}}{I_{ce}} = \frac{(12 - 6.9)\,V}{1\,\mu A} = 5.1\,K \]
Small signal performance

\[ q_m = \frac{T_{CA}}{n_V} = \frac{1 \text{ mA}}{26 \text{ mV}} = 38.5 \text{ mS} \]

\[ r_s = \frac{\beta}{q_m} = 100 \div 38.5 \times 10^3 = 2600 \Omega = 2.6 \text{ k}\Omega \]

\[ r_o = \frac{V_A}{T_{CA}} = \frac{50 \text{ V}}{1 \text{ mA}} = 50 \text{ k}\Omega \]

1) Voltage gain

\[ A_v = -q_m \frac{R_2 + R_o}{R_{S1} + R_{S2} + R_s} = -38.5 \times 10^3 \times \frac{5.1 \text{ k} + 50 \text{ k}}{2.6 \text{ k} \parallel 185 \text{ k} \parallel 0.15 \text{ k}} \]

\[ = -38.5 \text{ mS} \times 4.627 \text{ k} \times 0.9513 \]

\[ = -169.45 \]

2) Current gain

\[ A_i = -q_m \frac{R_o}{R_o + R_C} \]

\[ = -38.5 \text{ mS} \times \frac{50 \Omega}{50 \Omega + 5.1 \text{ k}} \times \frac{2.6 \text{ k} \parallel 185 \text{ k} \parallel 0.15 \text{ k}}{2.6 \text{ k} \parallel 185 \text{ k} \parallel 0.15 \text{ k} + 50 \Omega} \]

\[ = -38.5 \text{ mS} \times 0.9074 \times 1.0899 = -37.76 \]

3) Input impedance

\[ R_i = R_{S1} || R_{S2} = 1.0899 \text{ k} \]

4) Output impedance

\[ R_o = V_A || R_C = 4.627 \text{ k} \]