ECE 5115 - Lecture 24

So far, the assumptions that \( z_1, z_2, \) and \( z_3 \) are pure resistances. This is a close approximation for capacitors but inductors should be modeled with a series resistance along with inductance. However, the approach presented here still works. The only difference is that impedances are complex.

For example, consider Colpitts oscillators,

The impedance of the inductor may be modeled as

\[
z_3 = R + j\omega L
\]

Oscillations occur when

\[
(2 + 2 + 2)z_1 + (3 + 2 + 2 + z_1((2 + 2 + 2))) = 0
\]

Substituting \( z_1 = \frac{1}{jwC_1}, z_2 = \frac{1}{jwC_2}, z_3 = R + j\omega L \), the equation splits into two equations

\[
\begin{align*}
  1 & \left[ \omega L - \frac{1}{\omega C_1} - \frac{1}{\omega C_2} \right] - \frac{R}{\omega C_1} = 0 \quad \text{(imaginary part equals zero)} \quad (\star) \\
  \frac{1 + \frac{R}{\omega C_1}}{\omega^2 C_2} + \frac{1}{\omega C_1} \omega L & = 0 \quad \text{(real part equals zero)} \quad (\star \star)
\end{align*}
\]

If one defines \( C_1' = \frac{C_1}{1 + R/\omega L} \)

From (\star), we can solve for the resonant frequency.
\[ \omega_0 = \sqrt{\frac{L}{C_1 C_2}} \cdot \sqrt{\frac{1}{C_1 + C_2}} \]

Therefore, the resistance of the inductor changes the resonant frequency. The value of \( C_1 \) changes to \( C_1' \). Usually, \( r \ll \omega \), so the change of the oscillation frequency is usually small and since properties of inductor are known, this may be built into the design.

From \( \omega_0 \), one obtains the condition for oscillations:

\[ r \approx \frac{1}{2} \frac{1 + \beta}{\omega^2 C_2} - \frac{1}{C_1} \]

For oscillations to be sustainable:

\[ r \approx \frac{1}{2} \frac{1 + \beta}{\omega^2 C_2} - \frac{1}{C_1} \]

Notes:

1. It is desirable that \( \frac{1}{\omega^2 C_2} \) is large, i.e., \( C_1, C_2 \) are relatively small. However, they cannot be made extremely small. They need to be significantly larger than the parasitic capacitances of the transistors.

2. Excess highlights the importance of \( \beta \). If \( \beta \) is large the design is more robust and lower quality inductors may be used. Since \( \beta \) is frequency dependent, the oscillator design becomes more complex at higher frequencies.

Example 7.2: Design a Colpitts oscillator with \( f_0 = 5 \text{ MHz} \), \( L = 10 \mu \text{H} \) with unbaked \( Q_0 = 100 \). The transistor \( \beta \) is 100 as 100.

\[ Q_0 = \frac{\omega_0 L}{r} \]

\[ r = \frac{\omega_0}{\omega} \frac{L}{Q_0} = \frac{2 \pi f_0 L}{100} \]

\[ r = \frac{2 \pi \times 5 \times 10^6 \times 10 \times 10^{-6}}{100} = 314.12 \]
\[ \omega_0 = \left( L \frac{C_1 C_2}{C_1 + C_2} \right)^{1/2} \]

\[ \text{At } 5 \times 10^6 = \frac{1}{\sqrt{10^{16} \times C_e}} \Rightarrow C_e = \frac{C_1 C_2}{C_1 + C_2} = 100 \text{pF} \]

The easiest way to set \( C_1 \) \& \( C_2 \) is to assume \( C_1 = C_2 = 200 \text{pF} \).

Check of oscillation condition:

\[ RL \cdot R = \frac{1 + P_2}{\omega^2 C_2} - \frac{L}{C_1} \]

\[ 3.14^2 \cdot \text{Re} < \frac{1 + 100}{(2 \pi \cdot 5 \times 10^6)^2 \cdot (200 \cdot 10^{-12})^2} - \frac{10^{10}}{200 \cdot 10^{-12}} \]

\[ \text{Re} < 800 \text{kHz} \text{ (as long as Re is smaller than the calculated value the system will oscillate)} \]

So let us high draw the circuit:

\[ +V_G = 12 \text{V} \]

\[ R_E - 5 \text{d so } \text{Re} V_G = 0.1 V_G \]

Assuming bias current \( I_C = 1 \text{mA} \) \( R_E = 1.2 \text{k}\Omega \)
\[ V_{B} = V_{E} + V_{B EB(on)} = 1.2V + 0.65V = 1.85V \]

\[ V_{B} = \frac{R_{E1}}{R_{E1} + R_{E2}} \cdot V_{CC} = \frac{1}{1 + R_{E2}/R_{E1}} \cdot V_{CC} \]

\[ 1.85 = \frac{1}{1 + R_{E2}/R_{E1}} \times 12 \Rightarrow \frac{R_{E1}/R_{E2}} = 5.48 \]

\[ R_{B1}/R_{E2} = 5.48 \]

\[ R_{B1} = 10.15k \]

\[ I_{EB} > 100 \cdot I_{B} = 1mA \Rightarrow R_{E1} + R_{E2} = 12k \]

\[ V_{C} = \frac{V_{CC} - V_{C min}}{2} + V_{C min} = 2 + \frac{12 - 2}{2} = 2 + 5 = 7V \Rightarrow R_{L} = 5K \]

Finally, perform the adjustment at \( C \)

\[ R_{11} = \frac{R_{1}}{g_m} = \beta \cdot \frac{V_{C}}{J_{CV}} = 100 \times \frac{25uV}{1mA} = 2.6k\Omega \]

\[ C_{1}' = C_{1} \]

\[ C = C_{1}' \left( 1 + \frac{1}{g_{m} r_{11}} \right) = 200pF \left( 1 + \frac{314}{2.6 \times 10^{3}} \right) = 200.2pF \]

So, the final circuit diagram

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Note: The design is conducted in the following manner:

1) Select the oscillator configuration
2) Place the elements in the resonant path of the circuit
3) Adjust the biasing parameters
4) Verify the design.
Aplihide and phase stability

- Analysis so far focused on frequency of oscillation & damping of oscillation
- Amplitude of oscillation is not controlled.

Two ways of controlling the amplitude of oscillation

- Limiting oscillations
  - Oscillator seen as an unstable circuit
  - The amplitude of oscillation limited by the transistor gain parameter
  - When the oscillations reach some magnitude, the amplification of the transistor stage decreases and the amplitude of the oscillations stabilize

- Addition of extra circuits and circuit elements to control the amplitude of oscillations

Frequency and Phase stability

- Frequency of oscillation determined by the resonant circuits
- The more selective the resonant circuit is, the more stable the frequency of oscillation becomes.
  - Oscillator need a high quality resonant circuits (i.e. circuits with high $Q$)
  - It is easy to find high quality capacitors
  - Finding inductors with high $Q$ is not that easy.

Phase stability - Change of the oscillation frequency as a function of the total phase shift through the closed loop

Consider phase plots for the $G_H(j\omega)$ of the oscillator. The plots are given for two systems $G_H(j\omega), G_H(j\omega)$
Consider a parallel resonant circuit.

\[ V_0 = \frac{1}{jQ} \left( \frac{w_0}{w} - \frac{w}{w_0} \right) \]

\[ \frac{d}{dw} \left( \frac{V_0}{j} \right) = \frac{d}{dw} \left( -Q \left( \frac{w}{w_0} - \frac{w_0}{w} \right) \right) = \]

\[ = -Q \left( \frac{w^2 - w_0^2}{w_0^2 - w^2} \right) \]

When \( w = w_0 \),

\[ \frac{d}{dw} \left( \frac{V_0}{j} \right) = -\frac{2Q}{w_0} \]
We define phase stability factor as

$$S_p = \frac{1}{\omega_0} \cdot \frac{d(\Delta \phi)}{d(\Delta \omega)} \bigg|_{\omega = \omega_0}$$

In this case, $S_p = 20$

Therefore, the phase stability of the oscillator circuits depend on the $Q$ factor of the resonant components. Practical components have a $Q$ factors $\sim$ several hundreds. Much higher $Q$ factors are obtained using crystal oscillator circuits.

Electrical characteristics of piezoelectric crystals.

[Diagram of oscillator circuit with labeled components and text describing the circuit components and their roles in phase stability.]
For most practical crystals, \( \varepsilon_r \) is very small and \( \varepsilon_0 \) is high.

If \( \varepsilon_r \) is high, the equivalent impedance of the crystal may be expressed as

\[
\frac{1}{2 \sigma c_0} \left( \frac{\omega L_1}{1 + \omega^2 c_1} \right) = - \frac{(1 - \omega^2 L_1 c_1)}{\omega (c_1 + c_0)} \cdot \frac{1}{\omega} + \frac{j \omega L_1}{\omega} + \frac{1}{\omega} = - \omega^2 L_1 \left[ \frac{c_1}{c_1 + c_0} \right]
\]

The impedance becomes zero at some resonant frequency.

\[
\omega_{sr} = \frac{1}{\sqrt{\varepsilon_0 c_1}}, \quad \frac{1}{p_{sr}} = \frac{1}{2 \pi N L c_1}
\]

and it will become infinite at parallel resonant frequency.

\[
\omega_{pr} = \frac{1}{\sqrt{\frac{1}{\varepsilon_0 c_1} + \frac{\varepsilon_r}{c_0}}}, \quad \frac{1}{p_{pr}} = \frac{1}{2 \pi N L c_1 \sqrt{1 + \frac{\varepsilon_r}{c_0}}}
\]

Therefore, crystals may be used as either parallel or series resonant circuits of a very high \( \omega \).