Oscillators

- Electronic circuit producing periodic output function - usually no input is necessary
- Simplest - harmonic oscillators provide sinusoidal output function
- May be
  - Crystal controlled oscillators
  - Voltage controlled oscillators
  - Voltage controlled crystal oscillators

Oscillation theory - Conditions for oscillation

Oscillators - unstable feedback networks (positive feedback)

Consider a feedback network.

\[ V_0 = \frac{G(j\omega)}{1 + G(j\omega)H(j\omega)} \cdot V_i \] (1)

- For an oscillator, the output is non-zero even if the input \( V_i \) is zero. This can be possible only if the denominator of the transfer function becomes equal to zero.
- By setting the denominator of (1) to zero, one obtains a condition for oscillations given by

\[ 1 + G(j\omega)H(j\omega) = 0 \quad \text{or} \quad G(j\omega)H(j\omega) = -1 \]
which way be split into two conditions:

\[
|G(j\omega) H(j\omega)| = 1 \quad \text{&} \quad \angle [G(j\omega) H(j\omega)] = \pm \pi. \quad (**) \]

Since the primary purpose of the feedback network is to control frequency of oscillation, the conditions given in (**) would be satisfied only on a single frequency \( \omega_c \). The \( \omega_c \) becomes the frequency of oscillation.

**Example.** Consider the circuit given in the following figure.

If the biasing resistor \( R_B \) is assumed large, the feedback from collector to base is essentially broken and the circuit simplifies as

\[ R = (\kappa V_{c}) R_E \]
\[ \omega^2 R^2 (C_1+C_2)^2 \gg 1 \] (step up nonlinear) the circuit may be approximated as

\[ R_e = R \left( \frac{C_1+C_2}{C_2} \right) L \]

The forward gain is given by

\[ \frac{V_o}{V} = G(j\omega) = \frac{C_1}{C_1+C_2} = \frac{C_1}{C_1+C_2} \left( j\omega C + \frac{1}{j\omega L} + \frac{1}{R_e} \right)^{-1} = \left[ j\omega C - \frac{1}{j\omega} \right]^{-1} + \frac{1}{R_e} \]

where \( C = \frac{C_1C_2}{C_1+C_2} \)

The feedback ratio of the circuit

\[ \beta = \frac{V_o}{V_s} = \frac{C_1}{C_1+C_2} = H(j\omega) V_o \]

The condition for oscillations is that \( G(j\omega) H(j\omega) = +1 \) (positive feedback)

\[ |G(j\omega)| |H(j\omega)| = 1 \] (1)

\[ \times G(j\omega) + \times H(j\omega) = 0 \] (2)

From (2): Since \( \times H(j\omega) = 0 \) \( \times G(j\omega) = 0 \). This happens at the resonant frequency

\[ \omega_0 = \frac{1}{\sqrt{L/C}} \]

At that frequency \( 2L = \frac{1}{R_e} = \frac{R (C_1+C_2)^2}{1} R_L \)
and \( \text{gain} = g_m R \frac{e^{s+2}}{e^2} \)

From condition (1), one obtains,

\[
|\text{gain}| \cdot |H(s)| = g_m R \left( \frac{e^{s+2}}{e^2} \right) \cdot \frac{1}{s^2 + \frac{R}{g_m} s + \frac{1}{C_2}} = 1
\]

which may be used to determine values of the components and the biasing point of the oscillator.

**Example 7.1.** Use previous analysis to design an oscillator with \( f_c = 20 \text{ MHz} \) using common base transistors with \( f_t = 100 \).

Design process is looping through above equations using trial and error approach.

Assume biasing current \( I_e = 1 \mu A \), \( g_m = \frac{I_e}{V_t} = \frac{1 \mu A}{26 \mu V} = 0.038 \text{ S} \)

\[
R = \frac{1}{g_m} \left( \frac{1}{R_e} + \frac{1}{r_b} \right) \approx 26 \Omega \quad (R_e \approx 300 \Omega)
\]

Assuming \( V_{ce} \%

\[
|\text{gain}| \cdot |H(s)| = g_m R \left( \frac{e^{s+2}}{e^2} \right) \cdot \frac{1}{\frac{C_2}{C_1}} = g_m R \cdot \frac{e^{s+2}}{e^2}
\]

In oscillator design the loop gain is selected to be large, i.e., \( 1 > \), that way the circuit becomes unstable and the amplitude of oscillations starts increasing. As a result, the transistor starts saturating and the gain is reduced (\( f_t \) becomes reduced). In this example we set

\[
g_m R \cdot \left( 1 + \frac{C_2}{C_1} \right) = 3
\]

\[
0.038 \times 26 \left( 1 + \frac{C_2}{C_1} \right) = 3 \Rightarrow \frac{C_2}{C_1} = 2
\]

Let \( C_1 = 0.5 \text{nF}, \ C_2 = 1 \text{nF} \)

\[
C = \frac{C_1 \cdot C_2}{C_1 + C_2} = 0.333 \text{ pF}
\]

\[
L = \frac{1}{\omega^2 C} = \frac{1}{(2 \pi \times 20 \times 10^6)^2 \times 0.333 \times 10^{-12}} = 0.19 \mu \text{H}
\]
$RL \text{ needs to be large enough} \quad R_L \gg R \left( \frac{C_1 + C_2}{C_1} \right)^2$

$R_L \gg 26 \left( 1 + 2 \right)^2 = 234 \Omega$

$R_L = 2.3 \, k\Omega \quad \text{(set} \quad P_L = 10 \, R \left( \frac{C_1 + C_2}{C_1} \right) \right)$

$RE \gg R_e = 262 \quad \text{Set} \quad RE = 1.2 \, k\Omega \quad (10\% \text{ of} \quad Vcc \text{ goes for temp stabilization})$

Then one bus. $Vcc = \frac{1}{2} \times R_e \times I_E = \frac{I_E \times R_B}{1 \times 2} - I_E \times R_C - VBE(on) = 0$

$12V - \frac{1}{2} \times 1\times 2 - \frac{I}{100} \times R_B - 10A \times 12 - 0.65 = 0$

$R_B = 71.5 \, k\Omega \quad \text{(large resistance)}$

Therefore:

$P_E = 1.2 \, k\Omega$

$P_B = 71.5 \, k\Omega$

$C_s = C_B = 0.1 \, \mu F$

$R_L = 2.3 \, k\Omega$

$R_C = 3 \, k\Omega$

Note: In general case analysis of the oscillator circuit is complicated. Therefore, we usually examine one of several typical designs. Examples of such designs are Colpitts oscillator, Pierce oscillator and Hartley oscillator.
Circuit analysis

Sometimes, direct circuit analysis is simpler than the approach based on the block diagram—especially for a single transistor oscillating circuit.

Consider a generalized transistor oscillating circuit:

\[ \begin{align*}
(2_1 + 2_2 + 2_3) &= -2_1 - 2_2 \\
-2_1 &+ (2_1 + 2_2) = 0 \\
0 &= 0
\end{align*} \]

For the circuit to oscillate, the currents \( I_1, I_2, I_3 \) need to be non-zero even when \( v_i = 0 \). This is only possible if the determinant of the system is equal to zero. That is:

\[ (2_1 + 2_2 + 2_3) - 2_1 + \beta_2 2_2 = 0 \]

\[ (2_1 + 2_2 + 2_3) - 2_1 + \beta_2 2_2 + 2_1 (2_2 + 2_3) = 0 \]

To simplify the analysis, only the case \( 2_1 = 0 \) (real input impedance) will be considered. The capacitive component of \( 2_1 \) may be assumed
as part of 2.

Assume that \( z_1, z_2 \) and \( z_3 \) are purely reactive, then \( \Phi \) splits into two equations:

\[
\left( z_2 + z_3 \right) = 0 \quad \text{(imaginary part equal to zero)}
\]

\[
2_1 \left[ (1 + \beta) z_2 + z_2 \right] = 0 \quad \text{(real part equal to zero)}
\]

Since \( 2_1 = 0 \), \( (1 + \beta) z_2 + z_2 = 0 \) \( \Rightarrow \) \( z_2 \) and \( z_3 \) are reactances of different type.

\[
(1 + \beta) z_2 = -z_2 \quad (\star)
\]

Since \( \beta = 0 \), \( 1 + z_2 = (1 + \beta) z_2 = 0 \)

or

\[
2_1 = \beta z_2 \quad (\star \star) \Rightarrow z_2 \) and \( z_3 \) reactances of the same type

The general configuration from above may be used to generate several typical oscillator configurations.

1) Colpitts oscillator

![Diagram of Colpitts oscillator]

Generalized scheme

![Diagram showing electrical components]

Practical circuit realization (Reece oscillator)
2) Harley oscillator

Generalized schematic

Practical circuit realization of Harley's oscillator.