Example 4.3 (text).

Quantities involved: \( Q_u = 100 \)

If a lossy resistor is added, what is loaded \( Q_L \)?

Resonant Frequency
\[
\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-12} \times 10^{-6}}} = 10^8 \text{ rad/sec}
\]

Resistance of the inductor:
\[
Q_u = \frac{\omega_0 L}{R_L} \Rightarrow R_L = \frac{\omega_0 L}{Q_u} = \frac{10^8 \times 10^{-6}}{100} = 10.2 \Omega
\]

For frequencies near resonance, the resistance of the inductor may be approximated with a parallel resistance given by
\[
R = \frac{R_L^2}{\omega_0 L} = \frac{(10^8 \times 10^{-6})^2}{10^8 \times 10^{-6}} = 10^5 \Omega = 100 \text{ k}\Omega
\]

Therefore, the equivalent resistance in parallel to the LC circuit is
\[
R_{eq} = R_L || R_{10} = 100 \text{ k} || 10 \text{ k} = 5 \text{ k}\Omega
\]

The loaded \( Q \) factor
\[
Q_L = \frac{R_{eq}}{\omega_0 L} = \frac{5 \times 10^3}{10^8 \times 10^{-6}} = 50
\]

Therefore, addition of the "loss" stage reduced the \( Q \) factor of the parallel resonant circuit by 200 relative to the unloaded one.

Current through reactive components

For parallel resonant circuit current through reactive components is much bigger...
Than the current through resistor load.

For example, when the circuit is in resonance, the voltage across the resistor is given by

\[ V_0 = I \cdot R \]

The current through capacitor would be

\[ I_c = \frac{V_0}{\sqrt{j \omega C}} = j \omega C V_0 = j \omega C I \cdot R \]

However, since \[ \omega^2 = \frac{1}{LC} \Rightarrow \omega C = \frac{1}{\omega L} \]

Therefore,

\[ I_c = j \left( \frac{R}{\omega L} \right) I \Rightarrow |I_c| = Q \cdot I \]

The current through the inductive element is \( Q \) times larger than the current through the resistor at the resonant frequency.

Parallel resonant circuits including transformers.

Transformers are extensively used in lower frequency RF electronics to provide:

1. Phase inversion (reverters, modulators, mixers)
2. DC isolation
3. Impedance matching

Consider parallel resonant circuit.

\[ \text{At the resonant frequency, LC impedance becomes infinite - contributes to selectivity.} \]

4. The voltage at the output depends on \( R \).
   - If \( R \) is small, the voltage is small.
If \( R \) represents input resistance of the next stage, one would like it to be large. If it is not, then a method is needed to transform it.

Consider a transformer circuit:

\[
V_1 = j\omega L_1 I_1 + j\omega M I_2 \\
V_2 = j\omega L_2 I_1 + j\omega M I_2
\]

(We note: the figure in the text is incorrect.)

There are many equivalent circuits for the transformer. One of the most useful one is given by:

\[
V_1 = j\omega L_1 I_1 + j\omega M I_2 \\
V_2 = j\omega L_2 I_1 + j\omega M I_2
\]

where \( k = \frac{M}{\sqrt{L_1 L_2}} \)

Ideal transformer is a mathematical abstraction. The equations used to describe it.

Ideal transformer is given by (independent of frequency):

\[
\begin{align*}
V_1 &= N V_2 \\
I_1 &= -\frac{I_2}{N}
\end{align*}
\]

Since no power is dissipated in the transformer itself, one obtains:

\[
\begin{align*}
V_1 I_1 &= V_2 I_2 \\
I_1 &= -\frac{V_2}{V_1} I_2 - \frac{I_2}{N}
\end{align*}
\]

\[
\begin{align*}
\frac{V_1}{I_1} &= 2 L = \frac{N V_2}{I_2} = \frac{N^2 V_2}{I_2} = N^2 Z_L - \frac{2}{N}
\end{align*}
\]
Through the ideal transformer the impedance of the secondary is mapped into the primary circuit as $Z_2 = N^2 Z_1$. Since we usually are able to calculate $N$ the impedance seen by the source.

Parameters of the equivalent model

From primary \( V_f = j \omega (1-k^2) L_1 I_1 + j \omega k^2 L_1 (I_1 - I_2) \)

\[ = j \omega L_1 I_1 + \frac{j \omega k^2 L_1}{n} I_2 \]

For secondary $V_0 = \frac{k^2}{n} j \omega L_1 I_1 + \frac{j \omega L_1 k^2}{n^2} I_2 \quad (V_2 = V_f/n)$

Comparing (x=x) & (x) one sees that they become identical if

\[ \frac{k^2 L_1}{n} = M \quad \text{&} \quad \frac{L_1 k^2}{n^2} = L_2 \]

Solving for $n$ & $k$ one obtains

\[ n = k \left( \frac{L_1}{L_2} \right)^{1/2} \]

\[ k = \frac{M}{\sqrt{L_1 L_2}} \]

In practical circuit we have to make $k \ll 1$ (highly coupled transformers). In such scenarios the above formulation simplify even further with $k = 1$ and $n = (1/2)^{1/2}$.

The equivalent circuit becomes

![Equivalent Circuit](image-url)
Example 4.1. A highly coupled transformer: \( L_1 = 25 \mu H, \ L_2 = 400 \mu H \) is included in circuit below. Calculate the overall frequency response of the circuit.

\[
\begin{align*}
&L_1, \ L_2, \ C_1 = 2 \, \text{pf} \\
&\text{Draw equivalent circuit} \quad \kappa = \frac{L_1}{L_2} = 1:4 \\
&2_L = \frac{R_2}{R_1} \Rightarrow \quad \frac{P_2}{1 + j\omega L_2} \\
&n^2L_e = \frac{n^2P_2}{1 + j\omega \frac{C_1}{n^2}} \\
&n^2L_e \Rightarrow \quad R_{le} = n^2P_2, \quad C_{le} = \frac{C_1}{n^2} \\
&\text{Therefore the equivalent circuit seen by the input generator becomes.} \\
&Ce = \frac{C_1 + C_2}{n^2} = 8 \text{pf} + \frac{2 \text{pf}}{(0.2n)^2} = 40 \text{pf} \\
&\quad \Rightarrow \quad R_e = \frac{R_1}{n^2} = \frac{40k}{(0.2n)^2} = 25k \\
&\text{So the equivalent load seen by the power is a parallel resonant circuit with} \\
&\text{resonant frequency of} \\
&\omega_0 = \frac{1}{\sqrt{L_e C_e}} = \frac{1}{\sqrt{25k \times 40 \text{pf}}} \\
&\omega_0 = 5033 \text{ Hz}.
\end{align*}
\]
and a picker

\[ Q = \frac{R_{sw}}{r_a} = \frac{25 \cdot 10^3}{(2\pi \times 5 \times 10^6) \times (25 \times 10^6)} = 31.68. \]

The secondary voltage of this circuit is given as

\[ V_0 = \frac{V_i}{n} = 4.6V_i \]

Transducers with tuned secondaries

Consider an amplifier design given by:

What is seen by the transistor in the AC regime is a parallel resonant circuit with inductance equal to the primary and the secondary impedance transformed to the primary circuit. In the collector of the transistor
At a resonant frequency, the transistor sees pure resistance of $R_n$. The capacitor and inductor impedances cancel each other. By selecting proper $L$, the matching can be accomplished ( $V_L > 1$ impedance mismatch, $V_L < 1$ impedance matched). By selecting proper $C$, the operating frequency may be adjusted. By picking up primary inductor with appropriate $Q$, we have some flexibility of reducing $Q$ factor of the circuit (making it more broadband). $Q$ is bounded by the value of resistance mapped into the primary.

Other matching circuits:

Auto baluns (broadband)

* Easy way to change the impedance value by using a single inductor
* The inductor serves as an auto balun:

\[
\begin{align*}
V_1 &= j\omega I_1 L_1 + j\omega M (I_1 + I_2) + j\omega (I_1 + I_2) L_2 = j\omega (L_1 + L_2 + 2M) I_1 + j\omega (M + L_2) I_2 \\
V_2 &= j\omega (I_1 + I_2) L_2 + j\omega M I_1 = j\omega (L_2 + M) I_1 + j\omega L_2 I_2 (x)
\end{align*}
\]

Equivalent circuit:

\[
\begin{align*}
&\qquad (1-\kappa^2) L_1 = \frac{I_1}{(I_1+I_2)} \\
&\qquad (1-\kappa^2) L_2 = \frac{I_1}{(I_1+I_2)} \\
&\qquad (1-\kappa^2) R_L = \frac{I_2}{(I_1+I_2)}
\end{align*}
\]
Equations of the equivalent circuit are given by:

\[ V_1 = j\omega L_1 I_1 + \frac{j\omega L_2}{1} I_2 \]

\[ V_2 = \frac{1}{n} j\omega L_1 + \frac{1}{n^2} j\omega L_2 I_2 \quad (**) \quad (\text{See page 116}) \]

For (x) and (xy) to be equivalent

1. \( L = L_1 + L_2 + 2M \)
2. \( \frac{k^2 L}{n} = M + L_2 \)
3. \( L_2 = \frac{L}{n^2} \quad \Rightarrow \quad n = k \cdot \sqrt{\frac{L}{L_2}} \)

\( \Rightarrow (2) \quad \frac{k^2 L}{k \cdot \sqrt{L_2}} = M + L_2 \quad \Rightarrow \quad k = \frac{L_2 + M}{\sqrt{L_2}} \)

\[ n = \frac{L_2 + M}{\sqrt{L_2}} \sqrt{\frac{L}{L_2}} \]

Since \( L \) and \( L_2 \) are proportional to the square of number of turns, the equivalent turning ratio \( n = k \cdot \frac{N}{N_2} \) where \( N \) - total number of turns and \( N_2 \) number of turns in the by pass of the inductor. Since \( N > N_2 \), \( n \) is usually greater than 1 and the impendence seen is \( n^2 \) times larger. For that reason the tapped inductor autotransformer is usually referred to as the step up transformer. 

It increases the impedance seen by the circuit

HW 4.1, 4.2, 4.4, 4.7