Classical two port noise theory

* There are some parameters that are used to characterize noise behavior of general two port networks.
* These parameters are provided with circuit data sheets. Important to understand how they are defined.

Available source power

* Available source power — maximum power that the source can deliver to a matched load.

\[
2g = P_g + j X_g \\
2L = P_L + j X_L = P_g - j X_g \quad (i.e. \ 2L = 2g^*)
\]

Maximum power delivered to the load is

\[
P_{ac} = \frac{E^2 R_L}{4 P_g}
\]

Since \( g_{in} \) and \( P_g \) are properties of the source, the maximum available power is a property of the source only.

Available gain

Consider an arbitrary two-port network
Define

\[ \text{Pai} = \text{available power from the input source} \]

\[ \text{Pao} = \text{available power at the output of the two-port network} \]

Then, the available gain of the two-port network is defined as:

\[ \text{Gia} = \frac{\text{Pao}}{\text{Pai}} \]

In many cases, the available gain is a function of frequency. In such cases:

\[ \text{Gia} = \text{Ga}(f) = \frac{\Delta \text{Pao}}{\Delta \text{Pai}} \]

where \( \Delta \text{Pao} \), \( \Delta \text{Pai} \) - available powers in elementary bandwidth of frequency.

Transconductance Gain

Represents the ratio of the power delivered to the load and the available and available power of the source.

In the case when input and output are matched, the transconductance gain becomes the same as the available gain.

Equivalent noise bandwidth.

Consider a narrow two-port network with amplitude characteristics in a form of black filter. Assume that at the input of the two-port network one has a source of thermal noise. The available power at the output may be calculated as:

\[ \text{Pao} = G_o kT B_0 \]
In reality, every two port network has amplitude characteristics that is a function of frequency, i.e. \( G = G(f) \). For a small range of frequencies, one may write

\[
dP_{ao} = G(f) \cdot K T \cdot df
\]

The total power at the output is given by

\[
P_{ao} = \int_{f=0}^{f} dP_{ao} = \int_{f=0}^{f} G(f) \cdot K T \cdot df = K T \int_{f=0}^{f} G(f) \cdot df
\]

Comparing (x) and (xx), we can define the equivalent noise bandwidth as

\[
K T \cdot B_e \cdot G_{max} = K T \int_{f=0}^{f} G(f) \cdot df
\]

\[
B_e = \frac{1}{G_{max}} \int_{f=0}^{f} G(f) \cdot df \quad \text{equivalent noise bandwidth of a device}
\]

Example. Determine the equivalent noise bandwidth for the low pass filter given below.

\[
H(p) = \frac{1}{1 + \frac{1}{2 \pi f_c C}} \quad G(p) = \frac{1}{1 + \frac{1}{(2 \pi f_c L)^2}} , \quad G_{max} = 1
\]
\[ Be = \frac{1}{G_{\text{ave}}} \int f G(f)df = \frac{1}{L} \int \frac{d \left( \frac{211C2f}{1 + (211fC2)^2} \right)}{211C2} \]

\[ = \frac{1}{211C2} \text{atan} (x) \bigg|_{0}^{\infty} = \frac{1}{4C2} \]

For comparison purposes, \( B_{\text{dB}} = \frac{1}{211C2} \).

Note: \( B_{\text{dB}} \leq B_{e} \). The equivalent noise bandwidth is always smaller than the equivalent noise bandwidth. As the frequency response becomes more selective, the difference between becomes smaller.

**Equivalent noise temperature**

The available PSD of the noise source is given by

\[ P_{a}(f) = kT \]

The available power at the output of a noisy source

\[ P_{a} = kT \cdot B_{e} \text{ (Be - equivalent bandwidth)} \]

Therefore, one sees that the available power of the thermal source is entirely proportional to its temperature. This allows us to define the equivalent temperature of any noisy element. Consider the element with available noise PSD \( P_{a} \).

The equivalent temperature of the element is defined as the temperature of a thermal source that would produce the same available PSD.

\[ \frac{dP_{a}}{df} = P_{a} = kT_{\text{eff}} \]
It should be obvious that $T_{eff} \neq T$ (i.e., physical temperature of the device).

**Example 1**

\[
p_{\text{out}} = kT \cdot G(f) = kT_{\text{eff}}(f)
\]

Therefore $T_{\text{eff}}(f) = T \cdot G(f)$. If $G(f) > 1$, the equivalent temperature is larger than the physical temperature. $T_{\text{eff}}(f)$ is a function of the frequency.

**Example 2**

The antenna receives radiation from the space. The amount of this radiation changes as the noise delivered to the input resistance changes. However, the physical temperature of the antenna stays unchanged.

Consider now a two-port network. There are 2 sources of noise. The first is the noise delivered from the outside and the second one is noise generated at the network itself. The available power spectral density at the outside may be given as

\[
\frac{dP_{\text{out}}}{df} = G(\omega)kT + \frac{dP_{\text{self}}}{df}
\]

It is assumed that the outside noise and internal sources are independent.
One way interpret the second term in two ways:

1. The noise that would appear at the output if the input was noiseless.
2. The noise that appears at the output of the noise-free network due to an additional noise source. This source has PSD given by

\[ \frac{dP_{self}}{df} \]

However, one way write

\[ \frac{dP_{self}}{df} = \frac{k_{B}T_{e}c(f)}{\sigma_1^{2}(f)} \]

From where we find

\[ T_{e} = T_{eff}(f) = \frac{dP_{self}}{k_{B}G(f)} \quad \text{effective temperature of the self noise} \]

\[ \text{referenced at the system input} \]

Based on Kri's definition, one way write

\[ dP_{out}(f) = G(f) K_{B}(T + T_{eff}) = G(f) K_{B}T_{s} \quad (X) \]

\[ T_{s} \text{ - equivalent temperature of the network, reduced to the system input. It represents the temperature of the thermal source that would produce the same level of noise as the real system, when applied to the inputs of the noise-free version of the system.} \]

**Noise Factor, noise figure, sensitivity**

**Spot noise figure**

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Input noise PSD  \[ \frac{dP_{out}}{df} \quad \text{available output PSD} \]
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\[ k_{B}T_{o} \quad T_{o} = 290 \text{K} \]
The noise factor is the ratio of the available PSD at the output of the system and the available PSD from the input at standard temperature of \( T = T_0 = 290 \text{K} \):

\[
F(f) = \frac{dP_{in}}{df} = \frac{G_f}{G_0} \text{To} \tag{*}
\]

A noise factor is useful if the system is operating at temperatures close to \( T_0 = 290 \text{K} \). This is the case in most terrestrial systems.

2) A system temperature is used more frequently in cases when the noise source has a temperature that is significantly different than \( T_0 \). This is frequently the case in satellite communications.

According to (*), the noise factor is a function of frequency. In many applications, we define average noise factor as
\[ F = \frac{\int_{-\infty}^{+\infty} P_{a,0} \, dP_{a,0}}{\int_{-\infty}^{+\infty} G(\omega) \, d\omega} = \frac{P_{a,0}}{kT_0} = \frac{P_{a,0}}{kT_0 B_e} \]

where \( G_0 \) = maximum gain

\( B_e \) = equivalent bandwidth.

Usually \( F \) is expressed in dB, and when that is the case, it is referred to as the noise figure.

\[ F_{[\text{dB}]} = 10 \log (F) \]

Alternatively, the average noise factor (figure) may be calculated as

\[ F = \frac{\int_{-\infty}^{+\infty} F(\omega) \cdot G(\omega) \, d\omega}{\int_{-\infty}^{+\infty} G(\omega) \, d\omega} \]

Effective temperature of the cascaded two port networks.

Consider two cascaded two port networks.

\[ G_i \] - gain of the two port network

\[ F_i \] - noise factor of the two port network

\[ T_{ei} \] - effective temperature of self noise for the two port network
\[ \frac{dP_a}{df} = G_2 \left( G_1 k \left( T + T_{ei} \right) + k T_{e2} \right) \]

\[ = k G_2 \left( G_1 T + G_1 T_{ei} + T_{e2} \right) \]

\[ = \frac{G_1 G_2 k T + G_1 G_2 k \left( T_{ei} + T_{e2} \right)}{G_1} \]

The total available gain of the system is given by \( G = G_1 G_2 \).

\[ \frac{dP_a}{df} = G_1 k T + G_1 T_{es} = G_1 k (T + T_{es}) \]

where \( T_{es} = T_{ei} + \frac{T_{e2}}{G_1} \) - the equivalent temperature of self noise.

The same methodology may be extended to multiple cascaded two-port networks. By chain rule, one obtains

\[ T_{es} = T_{ei} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \ldots + \frac{T_{en}}{G_1 G_2 \ldots G_{n-1}} \]

The relationship in \((x-x)\) may be expressed using noise figures as well.

Since \( T_{ei} = T_0 (F-1) \), one may write

\[ T_0 (F_{es}-1) = T_0 (F_{i}-1) + T_0 (F_{2}-1) + \ldots + T_0 (F_{n-1}) \]

or

\[ F_{es} = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1 G_2} + \ldots + \frac{F_{n-1}}{G_1 G_2 \ldots G_{n-1}} \]

Equations \((xx)\) and \((xxx)\) are used to estimate the aggregate behavior of the system.
Example: Consider two cascaded noisy networks.

\[ \text{NF} = 2 \text{dB} \quad \text{NF} = 6 \text{dB} \]

\[ G_1 = 12 \text{dB} \quad G_2 = 10 \text{dB} \]

What is the overall noise figure? What is the total noise power at the output in \( B = 8 \text{kHz} \)?

\[ F_1 [\text{dB}] = 2 \text{dB} \quad F_1 = 10^{\frac{0.1 \times 2}{10}} = 1.59 \]
\[ G_1 [\text{dB}] = 12 \text{dB} \quad G_1 = 10^{\frac{0.1 \times 12}{10}} = 15.9 \]

\[ F_2 [\text{dB}] = 6 \text{dB} \quad F_2 = 10^{\frac{0.1 \times 6}{10}} = 4 \]
\[ G_2 [\text{dB}] = 10 \text{dB} \quad G_2 = 10^{\frac{0.1 \times 10}{10}} = 10 \]

\[ F_3 = F_1 + \frac{F_2}{G_1} = 1.59 + \frac{4}{15.9} = 1.78 \]

\[ F_3 [\text{dB}] = 2.5 \text{dB} \]

\[ P_i = kT F_3 B = 4 \times 10^{-18} \text{W} \times \frac{1.78}{2900} = 2.14 \times 10^{-14} \text{W} \]

\[ P_0 = G_1 G_2 P_i = 15.9 \times 10 \times 2.14 \times 10^{-14} \text{W} = 3.40 \times 10^{-12} \text{W} \]

\[ P_0 [\text{dBm}] = -114.68 \text{dBm} \]

Homework: 3.1, 3.2, 3.6, 3.8