Transmission line with generator and the load

\[ V(2) = V_0^* \left( e^{-j\beta_2} + P e^{j\beta_2} \right) \]

\[ I(2) = \frac{V_0^*}{Z_0} \left( e^{-j\beta_2} - P e^{j\beta_2} \right) \]

where

\[ V_0^* = \frac{V_g 2\eta}{2\eta + j\gamma} \frac{e^{-j\beta L}}{1 - j\eta P e^{-j2\beta L}} \]

and

\[ P_e = \frac{2\eta - 2\gamma}{2\eta + 2\gamma}, \quad P_g = \frac{2\eta - 2\gamma}{2\eta + 2\gamma} \]

Power delivered to the load

\[ P_e = \frac{1}{2} \text{Re} \left( V_{in} \frac{V_{in}^*}{2in} \right) = \frac{1}{2} \text{Re} \left( V_{in} \frac{V_{in}^*}{2in} \right) = \frac{1}{2} |V_{in}|^2 \text{Re} \left( \frac{1}{2in} \right) \]

\[ = \frac{1}{2} |V_g|^2 \left( \frac{2in}{2in + 2\gamma} \right)^2 \cdot \text{Re} \left( \frac{1}{2in} \right) \cdot (x) \]

Note: We have used previous result (page -18- of notes, ez (x)) which shows that the power flow along T_X does not depend on \( \gamma \). Therefore, the power delivered to the load is the same as the power delivered to the load.
Expression (*) may be simplified. Let \( 2m = \text{Real} \times n \) and \( 2g = \text{Real} \times y \)

Then (*) becomes

\[
P_L = \frac{1}{2} |V_g|^2 \left| \frac{\text{Real} + \text{Im}n}{\text{Real} + \text{Im}y} \right|^2 \text{Real} \left( \frac{1}{\text{Real} + \text{Im}n} \right)^2
\]

\[
= \frac{1}{2} |V_g|^2 \frac{\text{Real}^2 + \text{Im}n^2}{(\text{Real} + \text{Im}y)^2} \frac{\text{Real} \text{Real}}{\text{Real}^2 + \text{Im}n^2}
\]

\[
= \frac{1}{2} |V_g|^2 \frac{\text{Real}}{(\text{Real} + \text{Im}y)^2 + (\text{Im}n + \text{Im}y)^2} \quad \text{general result}
\]

Consider some special cases of significant interest.

**Case 1** Load impedance matched to the line \( 2L = 2o \), generator mismatched \( 2g \neq 2o \)

\[
2L = 2o \Rightarrow P_L = \frac{2L - 2o}{2L + 2o} = 0
\]

\[
\Rightarrow \text{VSWR} = \frac{1+|n|}{1-|n|} = 0
\]

\[
\Rightarrow 2m = 2o \cdot \frac{2L + i2o \text{Im}n}{2L - i2o \text{Im}n} = 2o
\]

The power delivered to the load

\[
P_L = \frac{1}{2} |V_g|^2 \frac{2o}{(2o + i2o \text{Im}n)^2 + (i2o \text{Im}n)^2} \quad (**)
\]

**Case 2** Generator matched to input impedance of the mismatched line

In other words:

\[
2g = 2m = 2o \cdot \frac{2L + i2o \text{Im}n}{2L - i2o \text{Im}n}
\]
Since generator is matched to the line, the reflection coefficient on the generator side of the line:

\[ p = \frac{2z - 2g}{2z + 2g} = 0 \]

Power delivered to the load:

\[ P_L = \frac{1}{2} |V_g|^2 \frac{R_g}{(R_g + 2g)^2 + (X_g + X_g)^2} = \frac{1}{2} |V_g|^2 \frac{R_g}{(2R_g)^2 + (2X_g)^2} \]

\[ P_L = \frac{1}{2} |V_g|^2 \frac{R_g}{4(R_g^2 + X_g^2)} \quad (\star) \]

Combining (\star) and (\star*), we see that (\star) may be smaller than (\star*). This means that for some values of load and generator impedances, it may be more beneficial to match the transmission line to the load instead of generator.

Example 1: Consider circuit with following parameters:

\[ 2g = 25 + j25 \]
\[ 2L = 50 \Omega \]

Determine which case delivers greater power to the load:

Case 1: Transmission line matched to the load
Case 2: Transmission line designed to match the 2L in the generator

Case 1: \[ 2a = 50 \Omega \text{ (matched to the load)} \]

\[ P_L = \frac{1}{2} |V_g|^2 - \frac{2a}{(2a + 2g)^2 + (X_g + X_g)^2} = \frac{1}{2} |V_g|^2 \frac{50}{(50 + 25)^2 + (25)^2} = \frac{1}{2} |V_g|^2 \times 0.008 \]

Case 2: TX line designed so that \[ 2g = 2 \text{ in} \]
Therefor in this case it is better to design transmission line that is matched to the load than the one that is matched to the generator.

Case 3. Transmission line design for wax power header. In this case we assume that Zg is fixed and we try to design transmission line to maximize power delivery to the load.

\[ P_L = \frac{1}{2} |V_g|^2 \frac{R_{in}}{(R_{in}+R_g)^2 + (X_{in}+X_g)^2} \]

\[ \frac{\partial P_L}{\partial R_{in}} = \frac{1}{2} |V_g|^2 \frac{R_{in} \left( (R_{in}+R_g)^2 + (X_{in}+X_g)^2 \right) - R_{in} \left( 2(R_{in}+R_g) \right)}{\left( (R_{in}+R_g)^2 + (X_{in}+X_g)^2 \right)^2} = 0 \]

or \[ R_g^2 - R_{in}^2 + (X_{in}+X_g)^2 = 0 \] (1)

\[ \frac{\partial P_L}{\partial X_{in}} = \frac{1}{2} |V_g|^2 \frac{2R_{in}(X_{in}+X_g)}{\left( (R_{in}+R_g)^2 + (X_{in}+X_g)^2 \right)^2} = 0 \]

or \[ X_{in}+X_g = 0 \] (2)

Solving (1) & (2) yields

\[ R_{in} = R_g \]

\[ X_{in} = -X_g \]

In this case transmission line is not matched to either load or generator, and as a result a standing wave is formed through the line. However, in this case the power delivery to the load is maximized.
The power delivered to the load is given as:

\[
P_L = \frac{1}{2} |V_g|^2 \frac{P_g}{(2P_g)^2 + (X_g - X_g)^2} = \frac{1}{2} |V_g|^2 \frac{1}{4P_g}
\]

**Example.** Consider the case depicted in Fig. 1

![Diagram of circuit](image)

Determine \(2m\) and \(2o\) of the transmission line so that the power transferred between generator and the load is maximized.

To maximize power transfer:

\[
2m = 2g^* = 25 - j5 \ \Omega
\]

\[
2m = 2_0 \frac{2_0 + j2_0 \tan(\beta L)}{2_0 + j2_0 \tan(\beta L)} = 2g^*
\]

let \(\tan(\beta L) = y\)

\[
2_0 (2_0 + jy2_0) = 2g^* (2_0 + jy2_0)
\]

\[
50(2_0 + jy2_0) = (25 - j5) (20 + jy50)
\]

\[
50(2_0 + jy2_0) = 2520 + 250y + j(1250y - 5200)
\]

By equating real and imaginary parts, one obtains...
2520 = 1250y \quad (1)
2a^2 = 1250y - 520 \quad (2)

From (1) \Rightarrow 2a = 10y. Substituting into (2)
100y^2 = 1250y - 520 \Rightarrow y = \sqrt{12} = 2N3

Solving for parameters of the line
2a = 250y = 250 \times 2N3 = 500N3 = 866.0252

\text{tan} \theta = \frac{2N3}{2N\bar{3}} \Rightarrow \theta = \text{atan}(2N3) = 1.29

Notes: 1) Even though this transmission line maximizes power transfer its implementation might be problematic due to high characteristic impedance.

2) In practice we try to avoid these kinds of matching and focus on circuit design. But maintaining certain characteristic impedance throughout the circuit typically we design generators/loads to have \( Z_g = Z_l = Z_0 = 50 \Omega \). This way we reduce power matching problems.

Homework 2.

Problems 2.4, 2.5, 2.6 and 2.7

ASK STUDENTS TO BRING COMPUTERS NEXT CLASS