Preparation for midterm exam

Topics:

1) Telegrapher's equations
2) Wave energy, reflection coefficient, VSWR, input impedance,
3) Lossless transmission line
4) Terminated line and power transfer, load matching
5) Smith chart calculations, impedance matching (Quarter wave transformer, shift)
6) Available noise power, Noise figure, Noise temperature
7) Nonlinear distortion
8) Calculation of noise figure, intercept point and dynamic range for cascaded systems

Test time: 1 hour 15 min
Test mode: open book/open notes
# problems: 4 - 5 problems

Review:
- Examples
- Sample problems
- HW problems
- Other problems in the book

Problem 1: Coaxial line has following characteristics at operating frequency of 1000 MHz

\[ R = 4 \Omega/m \quad L = 50 \mu H/m \]

\[ C = 45 \text{ nF/m} \quad \beta = 7 \times 10^{-4} \text{ S/m} \]

a) Calculate \( Z_0 \), \( \theta \), \( \phi \), \( V_p \), and \( \alpha \) at 1000 MHz

\[ Z_0 = \sqrt{\frac{R + jwL}{jwC}} = \left[ \frac{4 \times 1 + j \times 1000 \times 10^6 \times 450 \times 10^{-9}}{7 \times 10^{-4} + j \times 10^6 \times 100 \times 10^6 \times 50 \times 10^{-12}} \right]^{1/2} \]

\[ \approx 94.8683 + j 0.0386 = 95 \angle 0.0084 \text{ rad} \]
\[ y = \sqrt{(2 + j \omega L)(6 + j \omega C)} = \]
\[ = \left[ (4 + j 2\pi \times 10^3)(450 \times 10^{-9}))(7 \times 10^{-4} + j (2\pi \times 10^3)(50 \times 10^{-12} \right) \right]^{\frac{1}{2}}\]
\[ = 0.054 + j 29.8 \]
\[ \alpha = 0.054, \beta = 29.8 \text{ rad/m} \]
\[ V_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi \times 1000 \times 10^6}{298} = 2.11 \times 10^6 \text{ W/s} \]
\[ J = \frac{2\pi}{\beta} = \frac{2\pi}{298} = 0.21 \text{ W} \]

Problem 2: An antenna is connected to the TX through a TX line.

![Diagram]

The impedance of antenna: \( Z_a = 45 \Omega \)
Chlorodene's impedance: \( Z_0 = 50 \Omega \)
The generator impedance: \( Z_g = 50 \Omega \)
Power of the transmitter: \( P_g = 100 \text{ W} \)

Calculate:

a) VSWR at antenna port
b) The return loss at the input port
c) The reflected power at input

Note: Assume lossless line

\[ P_L = \frac{2Z_a - Z_0}{2Z_a + Z_0} = \frac{45 - 50}{45 + 50} = -0.0526 \]

\[ \text{VSWR} = \frac{1 + |P_L|}{1 - |P_L|} = \frac{1 + 0.0526}{1 - 0.0526} = 1.111 \]

\[ RL = 10 \log \left( \frac{P_{in}}{Pr} \right), \quad P_{in} = \text{input power} \quad Pr = \text{return power} \]
Problem 3. Consider the circuit shown in figure. Determine input impedance:

a) Using "manual calculations"

b) Using Smith chart

\[ RL = 20 \log \left( \frac{P_{in}}{P_d + P_{in}} \right) = 20 \log |P_{in}| \]

\[ = 20 \log (0.0526) = 25.6 \text{ dB} \]

c) \[ P_r = |P_{in}|^2 P_{in} = (0.0526)^2 \text{ 100W} = 0.2767 \text{ W} \]

\[ Y_1 = \frac{1}{2_1} = 0.021 + j0.012 \]

\[ 2_2 = -j \frac{2_1}{\tan(2\pi \times 0.7)} = -j \frac{50}{\tan(2\pi \times 0.7)} = -j16.246 \]
\[ Y_2 = \frac{1}{-j16} = j0.0616 \]

\[ Y_{tot} = Y_1 + Y_2 = (0.0185 + j0.0236) + j0.0616 = \]
\[ = (0.0185 + j0.0852) \Omega \]

\[ 2\beta = \frac{1}{Y_{tot}} = (2.48 - j1.12) \Omega \]

\[ Z_n = 2\beta z + j2\beta \tan(\phi_{ex}) = 2\beta + j2\beta \tan(\phi) \]
\[ = 50 \cdot \frac{(2.48 - j1.12) + j50 \tan(2\pi \times 0.2)}{50 + j(2.48 - j1.12) \tan(2\pi \times 0.2)} \]
\[ = (8.84 + j33.68) \Omega \]

b) Using smith chart (note: calculations in brackets - very numeric complicated above)

\[ Z_n = \frac{2\beta z}{z_0} = \frac{(7.5 + j25.5)}{50} = 1.5 + j0.5 \]

1) Draw \( Z_n \) point \( = 1.5 + j0.5 \)
2) Determine \( z_n \) by tracing towards generator by 6.7°

\[ 0.75 = 0.5 + 0.2 \theta \]
\[ 0.5 \theta \Rightarrow \text{full circle} \]
\[ 0.2 \theta \Rightarrow \text{radians (degrees towards receiver)} \]
\[ z_1 \Rightarrow 0.204 \]
\[ z_1 \Rightarrow 0.204 \times 0.2 \Rightarrow 0.404 \]

\[ z_1 = 0.71 - j0.42 \quad [ z_1 = 2z_0 = 38.5 - j21 \Omega ] \]
3) Determine \( y_i \) as a symmetric mapping of \( 2i \) through each origin. Also, \( 3i \) needs to be on the same YSWR circle.

From the Smith chart one reads: \( y_i = 10 \angle 0^\circ \)

\[
\left[ y_i = y_1, y_0 = y_1/20 = (10 + j0.6)/50 = 0.02 + j0.012 \right]
\]

4) \( z_\infty = \infty \) (open circuit) located on the right-hand side of the chart on the X-axis

Hence 0.7\( \angle \) towards generator and read \( z_\infty = -j0.32 \)

\[
\left[ z_\infty = z_0, z_0 = z_\infty/20 = -j0.32 \times 50 = -j1.6 \right]
\]

5) Admittance of the slab - map through chart center

\( y_2 = 1 \angle 31 \)

\[
\left[ y_2, y_0 = y_2/20 = 1/31/50 = 0.002 \angle 31 \right]
\]

6) Total admittance at point 1

\( y_1o = 10 + j0.6 + j31 \angle = 1.05 + j31.7 \)

7) Determine \( z_{10} \) - mapping being the center of the chart

\( z_{10} = 0.1 - j0.24 \)

\[
\left[ z_{10} = (0.1 - j0.24) \times 50 = 5 - j12 \right]
\]

8) Determine \( z_{10} \) by moving 0.2\( \angle \) towards open:

\( z_{10} = 0.85 + j1.55 \)

\[
\left[ z_{10} = z_{10} \times 20 = (17.5 + j17.8) \right]
\]
Problem 4. Consider the transmission line shown in the figure. Determine the power delivered to the load.

\[ V_g = 50 \text{V} \]
\[ Z_0 = 75 \text{Ω} \]
\[ Z_L = (45 + j25) \Omega \]
\[ l = 0.5 \lambda \]

\[ Z_{in} = \frac{2L + 2Z_0 \tan (\beta l)}{2L + j2L \tan (\beta l)} = 75 \frac{(45 + j25) + j75 \tan (0.5 \times 2\pi)}{75 + j(45 + j25) \tan (0.5 \times 2\pi)} \]

\[ Z_{in} = R_{in} + jX_{in} = 45 + j25 \Omega \]

\[ P_L = \frac{1}{2} |V_g|^2 \frac{R_{in}}{(R_{in} + 2Z_0)^2 + (X_{in} + X_g)^2} = \frac{1}{2} \left( \frac{45}{(45 + 50)^2 + (25 + 0)^2} \right) \]

\[ P_L = 5.83 \text{W} \]

Problem 5. Consider the situation shown in the picture.

\[ Z_0 = 50 \Omega \]
\[ 2L = 2.5 \Omega \]
\[ f = 2 \times 10^9 \text{Hz} \]
\[ P_{max} = 0.1 \text{ - maximum reflection coefficient} \]

(a) Determine \( Z_L \) so that the impedance at the load is matched to the impedance of the line.

(b) Determine the frequency range over which matching is acceptable.
a) \[ a = \sqrt{20.26} = \sqrt{25.50} = 3.586 \text{ m}\]

b) \[ 4f = f_0 \left[ 2 - \frac{4}{3} \cos^2 \left( \frac{\theta_{\text{max}}}{\sqrt{1 - \frac{2a}{2L}}} \right) \right] - \]

\[ = 2.10^3 \left[ 2 - \frac{4}{3} \cos^2 \left( \frac{0.1}{\sqrt{1 - 0.1^2}} \right) \right] \]

\[ = 7.34 \text{ MHz} \]

\[ \Delta f/f = 36.7\% \]

Problem 6: Calculate overall noise figure and gain in dB for the system shown in the figure.

\[ G_1 = 10 \text{ dB} \rightarrow 10 \]
\[ F_1 = 3 \text{ dB} \rightarrow 2 \]
\[ G_2 = -5 \text{ dB} \rightarrow 0.3162 \]
\[ F_2 = 4 \text{ dB} \rightarrow 2.51 \]
\[ G_3 = 20 \text{ dB} \rightarrow 100 \]
\[ F_3 = 2 \text{ dB} \rightarrow 1.58 \]
\[ G_4 = -1 \text{ dB} \rightarrow 0.79 \]
\[ F_4 = 1 \text{ dB} \rightarrow 1.26 \]

\[ F_T = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} \]

\[ = 2 + \frac{2.51 - 1}{10} + \frac{1.58 - 1}{10 \times 0.3162} + \frac{1.26 - 1}{10 \times 0.3162 \times 100} \]

\[ = 2.84 \rightarrow 3.4 \text{ dB} \]

\[ G_T = 10 + (-5) + 20 - 1 = 24 \text{ dB} \]
Problem 7. Calculate the overall $P_3$ power level of the system.

$$G_3 = 20 \text{ dB}$$

$$P_{81} = 16 \text{ dBm}$$

$$P_{82} = 16 \text{ dBm}$$

$$P_{B3} = 20 \text{ dBm}$$

Note: All $P_3$ points are referenced to the inputs of components.

$$\frac{1}{P_{3c}} = \frac{1}{P_{81}} + \frac{G_1}{P_{82}} + \frac{G_1 G_2}{P_{83}}$$

$$P_{81} = 10 \text{ dBm} \rightarrow 10 \text{ mW}$$

$$G_1 = 2 \text{ dB} \rightarrow 1.58$$

$$P_{82} = 16 \text{ dBm} \rightarrow 89.81 \text{ mW}$$

$$G_2 = -5 \text{ dB} \rightarrow 0.8162$$

$$P_{B3} = 20 \text{ dBm} \rightarrow 100 \text{ mW}$$

$$G_B = 20 \text{ dB} \rightarrow 100$$

$$P_{3c} = \left[ \frac{1}{10 \text{ mW}} + \frac{1.58}{89.81 \text{ mW}} + \frac{1.58 \times 0.8162}{100 \text{ mW}} \right]^{-1} = 6.91 \text{ mW} \rightarrow 8.4 \text{ dBm}$$

Output $P_3$ point

$$O_{P_{3c}} = P_{3c} \times G_t \Rightarrow$$

$$O_{P_{3c}} [\text{dBm}] = P_{3c} [\text{dBm}] + G_t [\text{dB}]$$

$$= 8.4 \text{ dBm} + 2 \text{ dB} - 5 \text{ dB} + 20 \text{ dB} = 25.4 \text{ dBm}$$
Problem 8. For the system shown in figure, calculate:

a) Overall system gain in dB
b) Overall noise figure in dB
c) Equivalent noise temperature in K

d) Minimum detectable power at input \((S/N)_{in} = 3\text{dB}\)
e) \(P_2\) levels (\(P_2\) & \(P_{31}\) in dBW)
f) Dynamic ranges \(DR_P\) and \(DR_f\)

\[
\begin{align*}
RF & \rightarrow \text{BPF} & G_1 = 10\text{dB} & \text{Lo = 1 GHz} & G_2 = 20\text{dB} \\
& \rightarrow \text{RF Amp} & F_1 = 2\text{dB} & \text{Lo = 1 GHz} & F_2 = 4\text{dB} \\
& \rightarrow \text{IF Amp} & P_{31} = 10\text{dBW} & h_c = 4\text{dB} & P_{32} = 13\text{dBW} \\
& \rightarrow 0.5 - 1.6\text{GHz} & & & \\
\end{align*}
\]

a) Overall system gain (everything is matched)

\[
G_T = -1\text{dB} + 10\text{dB} - 4\text{dB} + 20\text{dB} = 25\text{dB}
\]

b) \(F_T = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} \)

\[
\begin{align*}
F_1 &= 1\text{dB} \rightarrow F_1 = 1.126 \\
G_1 &= -1\text{dB} \rightarrow 0.79 \\
F_2 &= 2\text{dB} \rightarrow F_2 = 1.58 \\
G_2 &= 10\text{dB} \rightarrow 10 \\
F_3 &= 4\text{dB} \rightarrow F_3 = 2.51 \\
G_3 &= -4\text{dB} \rightarrow 0.40 \\
F_4 &= 4\text{dB} \rightarrow F_4 = 2.51 \\
G_4 &= 20\text{dB} \rightarrow 100 \\

F_T &= 1.126 + \frac{1.58 - 1}{0.79} + \frac{2.51 - 1}{0.79 \times 10} + \frac{2.51 - 1}{0.79 \times 10 \times 0.40} = 2.37 \\
F_T &= 10\log(2.37) = 3.75\text{dB}
\end{align*}
\]
c) Since everything is at room temperature

\[ F_T = 1 + \frac{1}{e/e_0} \Rightarrow e = e_0 (F-1) = 290 (2.37 - 1) = 397.6 \, K \]

d) \[ P_{3_1} = \infty \]
\[ P_{3_2} = 10dBm \rightarrow P_{3_2} = 10mW \]
\[ P_{3_3} = 12dBm \rightarrow P_{3_3} = 20mW \]
\[ P_{3_4} = 16dBm \rightarrow P_{3_4} = 39.81mW \]
\[ G_1 = -1dB \rightarrow 0.79 \]
\[ G_2 = +10dB \rightarrow 10 \]
\[ G_3 = -4dB \rightarrow 0.4 \]
\[ G_4 = 20dB \rightarrow 100 \]

\[ P_3 = \left[ \frac{1}{P_{3_1}} + \frac{G_1}{P_{3_2}} + \frac{G_1 G_2}{P_{3_3}} + \frac{G_1 G_2 G_3}{P_{3_4}} \right]^{-1} = \]
\[ = \left[ \frac{1}{0.79} + \frac{0.79}{10mW} + \frac{0.79 \times 10}{20mW} + \frac{0.79 \times 10 \times 0.4}{39.81mW} \right]^{-1} = \]
\[ = 1.817mW \rightarrow 2.56 \, dBm \]

\[ O_{P_3} = P_3 [dBm] + 6 \gamma [dB] = 2.56 \, dBm + 2 \, dB = 27.56 \, dBm \]

e) \[ S_{min} = 10 \log (K T B e) + F_T + (S/N)_{min} = \]
\[ = 10 \log \left( 4 \times 10^{-18} \frac{W}{Hz} \times 300 \times 10^3 Hz \right) + 3.75 \, dB + 3 \, dB \]
\[ = -110.24 \, dBm \]

f) \[ \frac{D_{P_3}}{P_{3_1 \, approx}} = \left( P_3 - 10 \right) - S_{min} = (2.56 - 10) - (-110) = 102.8 \, dBm \]

\[ D_{P_3} = \frac{2}{3} \left( P_3 - S_{min} \right) = \frac{2}{3} \left( 2.56 - (-110.24) \right) = 75.2 \, dB \]