Rain attenuation due to Rain

Most rain attenuation models use surface measured rain rate as a fundamental physical variable and predict rain attenuation using expression

\[ A_t[\text{dB}] = cR^b \cdot L(R) \] / The book uses \[ A_t[\text{dB}] = k_2 R^b \cdot L(R) \] as well.

\( R \) - rain rate in \( \text{mm/h} \)
\( b \) - frequency and polarization dependent parameters
\( L(R) \) - Effective path length (usually a complex function of \( R \))

Major rain attenuation models

ITU-R Model \( \rightarrow \) adopted by the textbook

- Crane Global Model
- Crane Two-Component Model
- DAH Model ("USA Model")
- Excel Rain Attenuation Model
- Manning Model

ITU-R Model. (ITU-R P.618-9 - posted on the website)

Approach is based on general formula

\[ A_t[\text{dB}] = k_2 R^b \cdot L(R) = b \cdot L(R) \]

The attenuation is calculated for 99.99% and then corrected to other link reliabilities.

The approach has 5 steps:

1. Calculate \( b \).
2. Steps 3, 8, 9 calculate \( L(R) \).
3. Finally, step 3 adjusts the attenuation to different link reliability requirements.
Input parameters:

- $R_{0.01}$ - point rainfall rate for 0.01% of an average year (mm/hr)
- $h_s$ - height above mean sea level at the rainfall station (km)
- $\Theta$ - satellite elevation angle (degrees)
- $\phi$ - latitude of the rainfall station (degrees)
- $f$ - frequency in GHz
- $R_e$ - effective radius of the Earth ($R_e=6370$ km)

Definitions:

- A - frozen precipitation
- B - rain height
- C - liquid precipitation
- D - Earth-Satellite path

Step 1: Calculating the rain height above mean sea level

$$h_r(\phi) = \begin{cases} 
5 - 0.075(\phi-23), & \phi > 23^\circ \\
5, & \phi \in [-21, 23]^\circ \\
5 + 0.1(\phi + 21), & \phi \in [-71, -21]^\circ \\
0, & \phi < -71^\circ 
\end{cases}$$

$\phi \in (-89.6^\circ, 89.6^\circ)$ - validity of the model
2. **Computation of slant angle pull.**

\[ L_s = \frac{h_t' - h_s}{\sin(\theta)} \text{ in km} \]

\[ L_s = \frac{2(h_t' - h_s)}{\left(\sin^2 \theta + \frac{2(h_t' - h_s)}{h_t} \right)^{1/2} + \sin \theta} \text{ in km} \]

Note: In most weanly of systems \( \theta \geq 5^\circ \)

3. **Calculate horizontal projection of the slant pull.**

\[ L_g = L_s \cdot \cos(\theta) \text{ in km} \]

4. **Obtain 20.01% rain rate.**

The 20.01% rain rate may be obtained from either climate map tables or exceedance curves. The values obtained from climate maps are listed as follows.

<table>
<thead>
<tr>
<th>Climate Zone</th>
<th>20.01%</th>
<th>Climate Zone</th>
<th>20.01%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8</td>
<td>L</td>
<td>60</td>
</tr>
<tr>
<td>B</td>
<td>12.5</td>
<td>M</td>
<td>63</td>
</tr>
<tr>
<td>C</td>
<td>15</td>
<td>N</td>
<td>9.5</td>
</tr>
<tr>
<td>D</td>
<td>19</td>
<td>P</td>
<td>145</td>
</tr>
<tr>
<td>E</td>
<td>22</td>
<td>Q</td>
<td>115</td>
</tr>
<tr>
<td>F</td>
<td>28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>42</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Rain rates are in mm/h
Step 5. Calculate specific attenuation $\mu_{0,0.01}$

$$\mu_{0,0.01} = k_0(n_0,0.01)^n$$ in dB/km

<table>
<thead>
<tr>
<th>Frequency</th>
<th>$k_0$</th>
<th>$n_0$</th>
<th>$\mu$</th>
<th>$\delta y$</th>
<th>$\delta y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.008630</td>
<td>0.000811</td>
<td>1.121</td>
<td>1.075</td>
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<tr>
<td>6</td>
<td>0.01175</td>
<td>0.001685</td>
<td>1.808</td>
<td>1.245</td>
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</tr>
<tr>
<td>8</td>
<td>0.034941</td>
<td>0.003195</td>
<td>1.127</td>
<td>1.210</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.0101</td>
<td>0.009827</td>
<td>1.276</td>
<td>1.264</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.0188</td>
<td>0.01688</td>
<td>1.264</td>
<td>1.260</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.0751</td>
<td>0.00611</td>
<td>1.099</td>
<td>1.065</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.127</td>
<td>0.167</td>
<td>1.021</td>
<td>1.030</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.236</td>
<td>0.213</td>
<td>0.989</td>
<td>0.929</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.536</td>
<td>0.471</td>
<td>0.873</td>
<td>0.858</td>
<td></td>
</tr>
</tbody>
</table>

Note: Strong dependency on frequency & mild dependency on polarization.

Step 6. Calculate horizontal reduction factor for 0.01% of the time

$$\nu_{0.01} = \frac{1}{1 + 0.78 \sqrt{\frac{L_{0,0.01}}{f} - 0.38 \left(1 - \exp(-21.81)\right)}}$$

Step 7. Calculate vertical reduction factor for 0.01% of the time

1. $\gamma = \arctan\left[\frac{h_0 - h_s}{L_{0,0.01}}\right]$ in deg.

2. for $\gamma > 0$

   $$L_D = \frac{L_{0,0.01}}{\cos(\gamma)} \text{ in km}$$

   else

   $$L_D = \frac{(h_0 - h_s)}{\sin(\gamma)} \text{ in km}$$
7.3 \( |P| < \theta \) \( x = 36 - |P| \) deg

\[ x = 0 \]

7.4. \( V_{001} = \frac{1}{1 + \sqrt{\sin(\theta)} \left[ 31 \left( 1 - e^{-6/\theta} \right) \frac{L_{p}}{\theta} - 0.45 \right]} \]

step 8. The effective path length is

\[ L_E = L_p \cdot V_{001} \text{ in km} \]

step 9. The predicted attenuation exceeded in 0.01% of an average year is given by

\[ A_{0.01} = V_{001} \cdot L_E \text{ in dB} \]

step 10. Adjustment for \( p \neq 0.01 \% \); Note: \( p \in (0.01 \% - 5 \%) \)

1. \( \frac{\theta}{\theta} \) \( p \geq 1 \% \) or \( |P| > 36^\circ \) \( \beta = 0 \)

2. \( \frac{\theta}{\theta} \) \( p < 1 \% \) and \( |P| < 36^\circ \) and \( \theta > 25^\circ \) \( \beta = -0.005 \left( |P| - 36^\circ \right) \)

else \( \beta = -0.005 \left( |P| - 36^\circ \right) + 1.8 - 4.25 \sin(\theta) \)

1.2. \( A_p = A_{0.01} \left( \frac{p}{0.01} \right) \left( 0.655 + 0.033 \ln(p) - 0.045 \ln(A_{0.01}) - \beta (1-p) \right) \alpha(1, \theta) \)

A spreadsheet is provided that implements the rain attenuation model in accordance with steps 1-10. Students need to become familiar with the spreadsheet.
Effect of rain attenuation

1) UL - on the uplink the rain introduces additional attenuation. The amount of attenuation at a given percentile is calculated using an ITU model (for example). This attenuation is used as minimum UL margin.

2) DL - on the downlink the rain introduces additional losses and changes the equilibrium temperature of the sky.

In clear sky - the system temperature may be calculated as:

\[ T_{S - cs} = \frac{T_{ce} + T_{ground} + T_{atmos} + \eta \cdot 250 (1 - 0.20 \times \%_a) }{T_A} \]

- \( T_{ce} \) - noise temperature of the front-end
- \( T_A \) - antenna's noise temperature.
- \( \eta \) - attenuation of clear sky (due to atmospheric...)

In a rainy sky - the system temperature may be calculated as:

\[ T_{S - min} = \frac{T_{ce} + T_{ground} + T_{atmos} + \eta \cdot 250 (1 - 0.20 \times \%_a) }{T_A} \]

\( \eta = \) coupling efficiency \( \approx 0.9 - 0.95 \)

Degradation of the DL \( C/N \) due to rain is given by:

\[ \Delta (C/N)_{DL} = (C/N)_{cs} - (C/N)_{min} = A_{min} + 10 \log \left( \frac{T_{S - min}}{T_{S - cs}} \right) \%
\]

The fade margin for DL needs to be set as \( \% \)

Note: Review distribution to link budget spreadsheets.