Problem: Consider transmission of a CW waveform. At the receiver point, the signal consists of two dominant multipath components. If the first component is taken as a reference, the second component is attenuated 6 dB, delayed by 1 μs and phase shifted by $-\frac{\pi}{4}$. The transmission frequency is 900 kHz.

a) What is the time domain representation of the received signal?

b) Determine the frequency response of the channel at a given frequency.

c) Assuming that the delay, attenuation and phase shift do not depend on frequency, determine the magnitude and phase response of the channel.

d) If instead of CW, the transmitter sends the signal with a BW of 30 kHz, determine if the channel may be assumed flat. The criterion of "flatness" is that the magnitude response of the channel over the signal spectrum does not change by more than 10%.

\[ Y(t) = A \cos(\omega_0 t) \]

\[ Y(t) = \frac{A}{2} \cos(\omega_0 t - \frac{\pi}{4}) \]

$\uparrow$

$\rightarrow$ Propagation delay

$\rightarrow$ Propagation delay, due to propagation

Amplitude is divided by 2. $\Rightarrow$ 6 dB power attenuation.
b) Transmitted signal

\[ s(t) = A \cos(\omega t) \]

Received signal

\[ y(t) = A \cos(\omega t) + \frac{A}{2} \cos \left[ \omega t - \omega_T \frac{T}{4} \right] \]

\[ = A \cos(\omega t) + \frac{A}{2} \cos(\omega t + \frac{\pi}{4}) + \frac{A}{2} \sin(\omega t + \frac{\pi}{4}) \sin(\omega t) \]

\[ = A \cdot \left[ \cos(\omega t) + \frac{1}{2} \sin(\omega t) \right] \]

\[ = A \sqrt{a^2 + b^2} \cos(\omega t - \theta) \quad \text{with} \quad \theta = \tan^{-1} \frac{1}{2} \]

\[ a = 1 + \frac{1}{2} \cos \left[ \pi \cdot 900 \cdot 1.6 \cdot 1.16^6 \cdot \frac{\pi}{4} \right] = 1.8536 \]

\[ b = \frac{1}{2} \sin \left[ \pi \cdot 900 \cdot (1.6 \cdot 1.16^6 + \frac{\pi}{4}) \right] = 0.3536 \]

Magnitude response \[ |H(\omega)| = \sqrt{a^2 + b^2} = 1.3990 \]

Phase response \[ \angle H(\omega) = -\tan^{-1} \frac{0.3536}{1.8536} = -0.2555 \]

\[ H(\omega) = 1.3990 \ e^{-j0.2555} \]
c) \[ s(t) = A \cos(\omega t) \]

\[ y(t) = A \cos(\omega t) + \frac{A}{2} \cos(\omega t - \frac{\pi}{4}) \]

\[ = A \cos(\omega t) + \frac{A}{2} \cos(\omega t) \cos(\frac{3\pi}{4}) + \frac{A}{2} \sin(\omega t) \sin(\frac{3\pi}{4}) \]

\[ = A \cdot \frac{1 + \frac{1}{2} \cos(\omega t + \frac{3\pi}{4})}{\sin(\omega t)} \]

\[ = A \cdot \sqrt{a^2(\omega) + b^2(\omega) \cdot \cos^2(\omega t - \alpha \tan \frac{b(\omega)}{a(\omega)})} \]

\[ = A \cdot H(\omega) \cos(\omega t - \alpha \cdot H(\omega)) \]

\[ H(\omega) = \sqrt{1 + \frac{1}{2} \cos(2\omega t + \frac{3\pi}{4})^2 + \left(\frac{1}{2} \sin(2\omega t + \frac{3\pi}{4})\right)^2} \]

\[ = \sqrt{1 + \cos(2\omega t + \frac{3\pi}{4})^2 + \frac{1}{4} \cos^2(2\omega t + \frac{3\pi}{4}) + \frac{1}{4} \sin^2(2\omega t + \frac{3\pi}{4})} \]

\[ = \sqrt{\frac{5}{4} + \cos(2\omega t + \frac{3\pi}{4})^2} \]

\[ 4 \cdot H(\omega) = -\alpha \tan \frac{\frac{1}{2} \sin(2\omega t + \frac{3\pi}{4})}{1 + \frac{1}{2} \cos(2\omega t + \frac{3\pi}{4})} \]

\[ d) H(\omega) = \sqrt{\frac{5}{4} + \cos(2\pi \omega + \frac{3\pi}{4})} \]

\[ H(\omega = 900) = \sqrt{\frac{5}{4} + \cos(2\pi \cdot 900 + \frac{3\pi}{4})} = 1.3990 \]

\[ H(\omega = 900 + 0.03) = \sqrt{\frac{5}{4} + \cos(2\pi \cdot 900 + \frac{3\pi}{4})} = 1.3461 \]

\[ \Delta H = 0.0529 \]

\[ \frac{\Delta H}{H(\omega = 900)} = \frac{0.0529}{1.3990} \approx 0.0372 \text{ or } 3.72\% < 10\% \]

Judgement is valid.
Problem 2. Consider a digital wireless communication system operating in a Rayleigh fading environment. The BW of the system is 30 kHz, and required SNR is 17 dB. The noise figure of the receiver is 5 dB.

a) Calculate the receiver sensitivity in dBm and mW.

\[ R_s = 10 \log \left( kT \eta B \right) + T + S/N \]

\[ = 10 \log \left( 4.15 \times 10^{-7} \right) + 20 \times 10^{-3} \times 2 \] + 8 + 17 = -104.2 \text{ dBm} \]

\[ R_s \text{ Sens} = 10^{1.5 \text{ dBm}} = 379.16 \text{ mW} \]

b) \[ \text{RSL} = -100 \text{ dBm} \implies P_{\text{in}} = 10^{0.1 \times (-100)} = 10^{-10} \text{ mW} \]

For a Rayleigh fading signal, the power is distributed exponentially. That is:

\[ P_n \sim \text{PDF}(p) = \frac{p}{P_{\text{in}}} \cdot \exp(-\frac{p}{P_{\text{in}}}) \]

Probability of the power falling below a given threshold, \( M \) is given as:

\[ P_{\text{PDF}} \leq M = \int_0^M \text{PDF}(p) dp = 1 - \exp\left(-\frac{M}{P_{\text{in}}}ight) \]
when $\gamma = 2 \times 5 \times 10^9$

\[
\Pr \{ P \leq R \times \text{Spec}^{\gamma} \} = 1 - \exp\left( - \frac{R \times \text{Spec}^{\gamma}}{\text{Pap}} \right) = 1 - \exp\left( - 3.79 \times 10^{-11} / 10^{-10} \right) = 0.82
\]

c) Noise Floor of the Rx is given by

\[
\text{NFE}_{\text{dB}} = 10 \log \left( 10^3 \frac{18 \text{W} \text{Hz}}{4.15} \right) \quad 20 \times 10^3 \text{Hz} + 8
\]

\[
= -121.2 \text{ dBm}
\]

\[
\text{NFE} = 7.57 \times 10^{-13}
\]

\[
\Pr \{ P \leq \text{NFE}^{\gamma} \} = 1 - \exp\left( - 7.57 \times 10^{-13} / 10^{-10} \right) = 0.9975
\]

Problem 5: Consider a digital system with (BER, SNR) relationship given by

\[
\text{BER} = 2 \exp(-5 \text{SNR})
\]

where SNR is the signal to noise ratio expressed in linear form.

The bandwidth of the system is 30 kHz and the RX noise figure is 8 dB. If the average EIRP of the signal is -90 dBm and the system is operating in Rayleigh fading environment, estimate the average BER. The power of the noise may be assumed constant.

\[
\text{NF} = 10 \log \left( 10^3 \frac{18 \text{W} \text{Hz}}{4.15} \right) \quad 20 \times 10^3 \text{Hz} + 8 = -121.2 \text{ dBm}
\]

\[
\text{NF} = 7.57 \times 10^{-13} \text{ W/Hz}
\]
The problem is given by

\[ P_{eq} = 15.9 \cdot 10^{-6} \]

The outage SNR is given by

\[ SNR_o = \frac{15.9 \cdot 10^{-6}}{7.37 \cdot 10^{13}} = 1320.74 \]

Therefore the PDF of the SNR is given by

\[ f_{SNR} = \frac{1}{SNR_o} \exp\left(-\frac{SNR}{SNR_o}\right) \]

The inverse BER can be calculated as

\[ BER_{in} = \int_0^{\infty} BER(SNR) \cdot f_{SNR}(SNR) dSNR \]

Let \( x = SNR \), \( x_o = SNR_o \)

\[ BER_{in} = \int_0^{\infty} 2 \exp(-x) \cdot \frac{1}{x_o} \exp\left(-\frac{x}{x_o}\right) dx = \]

\[ = \frac{2}{x_o} \int_0^{\infty} \exp\left(-\frac{x + 1}{x_o}\right) dx = \]

\[ = \frac{2}{x_o} \left. \exp\left(-\frac{x + 1}{x_o}\right) \right|_0^\infty = \]

\[ = \frac{2}{1320.74 + 1} = 0.0015 = 0.15\% \]

\[ = 0.15\% \]
Problem 4: Consider a 2X capable of detecting 400 instantaneous power measurements per second on a single frequency. The return time between two different frequencies is 1 ms. If the measurements are performed in 100 MHz frequency band and the maximum vehicle speed is 35 mph, determine the maximum number of frequencies that the receiver can use so that the data is collected in accordance with the criteria.

Total time required for a power measurement at a given frequency:

\[ T_t = T_w + T_r \]

\[ = \frac{1}{1000000} + 1 \text{ ms} = 0.00035 \text{ sec} \]

The number of measurements in scanning mode:

\[ N_w = 285.7 \text{ measurements/sec} \]

For 100 MHz, the airspeed distance is:

\[ \lambda = 40 \times \frac{5}{3} = 40 \times \frac{5}{3} / \text{1000000} = 6.31 \text{ m} \]

Time required to cover the distance of 40 \( \lambda \) at maximum speed:

\[ T = \frac{6.31 \text{ m}}{35 \times 1609 / 3600 \text{ m/s}} = 0.4084 \text{ s} \]

The number of measurements per 40 \( \lambda \):

\[ N_{40} = 0.4084 \times 285.7 = 115.2 \text{c} \]
Since the minimum number of measurements required per day is 50, the receiver can measure only 2 channels.

Problem 5. Consider a receiver operating space diversity with 2 diversity branches. The receiver sensitivity is -105 dBm and the average signal level is -95 dBm. The Rx is operating in the marginal fading environment.

1) Find probability of the signal falling below the sensitivity in one of the branches.

b) Calculate probability of the signal falling below the sensitivity in both branches simultaneously.

c) If the receiver is using selection diversity combining, calculate the improvement in average SNR.

1) \( P_{s, RSL < R_{x, Sens}} = 1 - \exp\left(-\frac{R_{x, Sens}}{R_{SL, dB}}\right) = \)

\[ = 1 - \exp\left(-\frac{10^{-0.1 \times 10}}{10^{-0.1 \times 9.5}}\right) = 1 - \exp\left(-\frac{3.16 \times 10^{-10}}{3.16 \times 10^{-10}}\right) \]

\[ = 0.0952 = 9.5\% \]

b) \( P_{s, RSL_1 < R_{x, Sens}} \& RSL_2 < R_{x, Sens} = 0.0952 \times 0.0952 = 0.0091 \)
9) $T_{SW} = 1.2\text{ps}$

10) $T = \frac{1}{1 + \frac{1}{2} + \frac{1}{2}} \times 1.2 = 0.4\text{ps}$

b) $T = 1 + \frac{1}{2} + \frac{1}{2} = 1.5\text{ps}$

11) What is the excess delay spread for $\Delta f$?

12) $\Delta f = \frac{1}{\frac{1}{2}} = 2\Delta f$

13) $A_{SW} = \frac{1}{k} = \frac{1}{2} = \frac{1}{3} 
\Rightarrow 1.76\text{dB}$

14) Fast time-harmonic, the signal waveform is given by...
Problem 7. Consider a microwave link operating in Orlando, FL. The parameters of the link are as specified in Figure below.

![Figure](image)

Estimate the reliability of the line.

\[ R = 100 \left( 1 - \frac{\text{Failure}}{365 \times 24 \times 60 \times 60} \right) \]

\[ \text{Failure} = \text{Togue} + \text{Train} = \text{Typ} + \text{Train} \]

\( a) \quad \text{Typ} = 0.4 \times f \times T \times D^2 \times 10^{-9} \)

\[ \text{ERP} = 40 \text{dBm} - 3 \text{dB} + 25 = 62 \text{dBm} \]

\[ P_{\text{micr}} = 10 \log \left( 4 \times 10^{-18} \frac{\text{W}}{\text{Hz}}, 10^2 \text{Hz} \right) + 5 \times 15 = -94 \text{ dBm} \]

\[ \text{DSL} = -94 \text{ dBm} + 3 \text{dB} - 10 \text{ dB} = -101 \text{dBm} \]
\[ F_{SL} = 96.6 + 20 \log(14) + 20 \log(0.5) = 133.50 \] 
\[ F_{SL} = E_{hp} - F_{SL} = 62 \text{dBW} - 133.50 = -71.50 \text{ dBW} \] 
\[ f_{H} = -71.50 \text{ dBW} - (10 \text{ dBW}) = 29.49 \text{ dB} \]

\[ T_y = 0.4 \cdot 14 \cdot 70 \cdot (5 \cdot 10^{-3}) \cdot 10^{-29.49/10} \]
\[ = 337.55 \]

b) Train

The maximum load that can be sustained by the link is given by

\[ R_t = \left[ \frac{40 + 40 \cdot 1.609}{90 \cdot 0.5 \cdot 1.609} \right]^{1/6} \]

For 14 Gb/s, \( a = 0.85 \cdot 10^{-2} \)

\[ b = 1.128 \]

\[ R_v = \left[ \frac{40 + 4.5 \cdot 1.609}{90 \cdot 0.35 \cdot 1.62 \cdot 0.5 \cdot 1.609} \right]^{1/6} = 888.95 \]
For Print.

1) time: 5-7:15 (2 hours and 15 min)
2) last week: open book, open notes
3) materials:
   * multipath propagation (1)
   * Rayleigh fading (1)
   ✓ * hee Criteria (1)
   * Diversity combining (1)
   ✓ * Channel sounding (PPM) (1)
   * N-wire link reliability
4) # problems - 5