RF Propagation (Lecture 12)

'Diversity combining

' Diversity combining - algorithm for combining of different diversity paths

' Usually combining is done at a common question, i.e., the output signal is

' produced as a linear combination of signals

' The combination may be done at RF (predetection combining) or

' at the baseband (postdetection combining)

' From the standpoint of the question of the output signal where the

' combining is done is not important

' Performance criteria at the combining method is either improvement in S/N or

' SRR

' Three basic forms of combining

' 1) Selection diversity / Scanning diversity

' 2) Maximum gain combining

' 3) Optimum gain combining

' We will analyze improvement from all three of the above diversity combining

' methods assuming Rayleigh fading statistics of the channel

' The analysis will be presented in the treatment of space diversity. This

' is done without a loss in accuracy of obtained results.
Selecting diversity

\[ Y = \ldots Y \]

Logic produces selection at

The best perform bands.

4. Signal in each diversity branch follows Rayleigh fading channel.

5. There is enough overlap between bands (interchannel) so that the signals are uncorrelated.

The envelope of the signal in the i^{th} branch follows PDF given by

\[ r_i \sim \text{pdf}(r_i) = \frac{k_i}{6} \exp\left( -\frac{r_i^2}{2\sigma_i^2} \right), i = 1, \ldots, M \]

Since the selection is done on the basis of \( S_i / \sigma_i \), or max \( S_i / \sigma_i \).

It is more important to consider distribution of the signal power.

Let \( S_i = \frac{k_i^2}{2} \) be the signal power. Then the S/N in each branch is

\[ R_i = \text{pdf}(R_i) = \frac{1}{\sigma_i^2} \exp\left( -\frac{R_i^2}{2\sigma_i^4} \right), i = 1, \ldots, M \]

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What \( R_i \) is the average signal power in a given small area.

In a small area far from the point of the main lobe constant and given as \( N \). The average S/N is \( \frac{R_i}{\sigma_i^2} = S_i \).
The probability of \( Y_i \) in the \( i \)th grade being less than a given threshold \( y_i \) is given by

\[
\Pr(Y_i \leq y_i) = \int_{-\infty}^{y_i} f(y_i|\xi_i) \, dy_i = \int_{-\infty}^{y_i} \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{(y_i - \xi_i)^2}{2\sigma_i^2}\right) \, dy_i = 1 - \exp\left(-\frac{y_i^2}{\sigma_i^2}\right)
\]

Since knowing the \( i \)th grade, the probability of all children falling below some threshold simultaneously is given by

\[
\Pr(Y_1, Y_2, \ldots, Y_n \leq y) = \left[1 - \exp\left(-\frac{y^2}{\sigma^2}\right)\right]^n = F\left(\frac{y}{\sigma}, n\right)
\]

Let us evaluate (4) for some specific value of \( y_n \)

If \( y = 0 \) (adequate size), \( F\left(\frac{y}{\sigma}, n\right) = F(0, n) = (1 - \exp(-1))^n \)

- \( n = 1 \) \( F(0, 1) = 0.6321 \)
- \( n = 2 \) \( F(0, 2) = 0.3916 \)
- \( n = 3 \) \( F(0, 3) = 0.2596 \)
- \( n = 4 \) \( F(0, 4) = 0.1597 \)

Following the same methodology, we obtain the following table.
Due to anomaly, the severity of fading is reduced. We see that even for two towers, almost never falls below 20 dB. The probability of fading to 10 dB below the mean is just about 14.
Reduction in fading may be seen at some time of the game. The mean of the output of selection receiver can be seen as

$$E_s = \int_{-\infty}^{\infty} p_s(x^2) dx^2$$

$$p_s(x^2) = \frac{d}{dx^2} \exp\left(-\frac{x^2}{\sigma^2}\right)$$

$$= \frac{1}{\sigma^2} \left[1 - \exp\left(-\frac{x^2}{\sigma^2}\right)\right]^{1/2} \cdot \exp\left(-\frac{x^2}{\sigma^2}\right)$$

Therefore

$$E_s = \int_{-\infty}^{\infty} \frac{x^2}{\sigma^2} \cdot \exp\left(-\frac{x^2}{\sigma^2}\right) \cdot \left[1 - \exp\left(-\frac{x^2}{\sigma^2}\right)\right]^{1/2} \cdot \exp\left(-\frac{x^2}{\sigma^2}\right) dx^2$$

Need to verify steps in between

$$= \frac{1}{\sigma^2} \sum_{k=1}^{\infty} \frac{1}{k}$$

(Dasgupta, pg. 388)

The average signal to noise improvement is given by

$$\Delta \text{SNR} = \frac{1}{\sum_{k=1}^{\infty} \frac{1}{k}}$$

<table>
<thead>
<tr>
<th>k</th>
<th>\Delta \text{SNR}</th>
<th>\Delta \text{SNR} (dB)</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>1.76</td>
<td>1.76</td>
</tr>
<tr>
<td>3</td>
<td>1.82</td>
<td>2.63</td>
<td>0.87</td>
</tr>
<tr>
<td>4</td>
<td>2.08</td>
<td>3.14</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Every new antenna is 3-5% less significant.
Maximum gain combining

\[ P_R = \frac{1}{N} \sum_{k=1}^{N} a_k R_k \quad \text{received signal} \]

\[ P_N = \frac{P_R^2}{2} = \frac{1}{2} \left( \sum_{k=1}^{N} a_k R_k \right)^2 \quad \text{power of the received signal} \]

\[ \text{Non} = \left( \sum_{k=1}^{N} a_k^2 \right) \cdot N \quad \text{power of the local noise at the receiver} \]

Therefore, the signal to noise ratio is given as:

\[ \gamma_s = \frac{P_R^2}{2 \text{Non}} = \frac{1}{2N} \left( \sum_{k=1}^{N} a_k R_k \right)^2 \]

\[ = \frac{1}{2N} \left( \frac{1}{2} \left( \sum_{k=1}^{N} a_k^2 \right) \right)^2 \]

Maximum gain combining was wanted (goal) so that $\gamma_s \to \infty$ is maximized. More elaborate analysis shows that

\[ a_k \sim \frac{P_R}{N} \]

Therefore, $\gamma_s \to \infty$.
\[
\chi^2 = \left( \sum_{k=1}^{N} \frac{y_k^2}{N} \right)^2 \quad \frac{1}{N^2} \left( \sum_{k=1}^{N} y_k^2 \right)^2 = \sum_{k=1}^{N} \frac{y_k^2}{N} = \sum_{k=1}^{N} \frac{y_k^2}{y_k^2}
\]

where \( y_k \) is the signal to noise ratio in the \( k \)th band.

The PDF of the aggregate SNR:

\[
y_k \sim \text{pdf}(y_k) = \left( \frac{y_k}{y_o} \right)^{M-1} \frac{y_k^{M-1}}{(M-1)!} \exp \left( -\frac{y_k}{y_o} \right)
\]

\[
C_\text{df}(x_k) = 1 - \exp \left( -\frac{x_k}{y_o} \right) \left( \frac{x_k}{y_o} \right)^{k-1} \quad \frac{y_k}{(y_k - 1)^2}
\]

<table>
<thead>
<tr>
<th>( y_k/y_o )</th>
<th>( M = 1 )</th>
<th>( M = 2 )</th>
<th>( M = 3 )</th>
<th>( M = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>63.21</td>
<td>26.62</td>
<td>2.03</td>
<td>1.90</td>
</tr>
<tr>
<td>-10</td>
<td>4.52</td>
<td>0.47</td>
<td>0.02</td>
<td>0.0004</td>
</tr>
<tr>
<td>-20</td>
<td>1.00</td>
<td>0.005</td>
<td>( \leq 0 )</td>
<td>( \leq 0 )</td>
</tr>
<tr>
<td>-30</td>
<td>0.1</td>
<td>( \leq 0 )</td>
<td>( \leq 0 )</td>
<td>( \leq 0 )</td>
</tr>
</tbody>
</table>

De Pois with \( M = 2 \) probability of deep Pois ( \( \geq 100^d \)) is negligible.

\[
\chi^2 = \chi^2 \quad \Rightarrow \text{improvement} = 1
\]

\[
\text{improvement in SNR}
\]

\[
\chi^2 = \chi^2 = \chi^2 \quad \Rightarrow \text{improvement} = 2
\]

\[
\chi^2 = \chi^2 = \chi^2 \quad \Rightarrow \text{improvement} = 3
\]

\[
\chi^2 = \chi^2 = \chi^2 \quad \Rightarrow \text{improvement} = 4
\]

\[
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\]
Equal gain combining

\[ Y_{\text{eq}} = \sqrt{N} Y_{\text{rx}} \]

\[ P_{\text{eq}} = \frac{P_{\text{rx}}}{N+1} \quad \text{output signal-to-noise ratio} \]

Treatment of equal gain combining is complicated analytically. Usually, it is done through computer simulation. The bank does not give the curves; copy from Jakes. (Need to get a copy of Jakes)

The curves of \( E_{\text{oc}} \) are within 0.5 dB at the maximum ratio.

\[ P_{\text{eq}} = P_{\text{rx}} \left( 1 + \frac{(N-1)^2}{8} \right) \]

\[ \begin{array}{c|c|c|c|c|c|c}
\hline
N & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
\text{Improvement} & 1 & 0.5 & 0.25 & 0.125 & 0.063 & 0.031 \\
\text{dB} & 2 & 2.5 & 3 & 4 & 5 & 6 \\
\hline
\end{array} \]
Consider a bundle of fibers the path of the aggregate signal for different counting methods are given as:

\[ \text{SC: } P(t) = \left(1 - e^{-\lambda t}\right)^2 \]

\[ \text{MRC: } P(t) = \frac{1}{e} \left(1 - e^{-\lambda t}\right) \]

\[ \text{FBC: } P(t) = \frac{1}{e} \left(1 - e^{-\lambda t}\right) \left[\text{FBC} \left(\frac{t}{\lambda}\right)\right] \]

Note:
1. The decay reduces from recombination at the source is large.
2. Differences in the fiber reduction between the counting methods are negligible.
3. Use of more than 2 antennas is rarely justifiable.
4. Using different weights, Finding Primary channel into AN/GN.