Lecture 3 RF propagation

Propagation over a reflecting surface

* Free space propagation occurs only under very restricted conditions
* Usually one finds many obstacles between TX & RX
  - Obstacles reflect the radio wave = Signal at the receiver is a superposition of these multiple reflections.
* The simplest case to analyze is when the TX and RX antenna are elevated, within line of sight from each other and above solid Earth.

The reflection coefficients of the earth

![Diagram of reflection](image)

* Plane which is perpendicular on the reflecting surface and which contains the direction of the wave propagation is called incident plane.
* Every reflected wave can be seen as a superposition of two waves.

1) One wave E reflected within incident plane - vertically polarized
2) E reflected normal on the incident plane - horizontally polarized
\[ W = \vec{E} \times \vec{H} \]

Reflects off horizontally polarized
radio wave.

Reflects off vertically polarized
radio wave.

At the point of incidence the EM wave must satisfy the boundary conditions.

- Of a great practical interest is situations when \( \Theta = \) air and \( \Theta \) represents the Earth (soil).
- The Earth can be characterized with a dielectric constant \( \varepsilon \) and its conductivity \( \sigma \).
- The reflection coefficient for two polarizations are given as

\[ S_H = \frac{E_{rH}}{E_{iH}} = \frac{\sin \psi - \sqrt{\frac{\varepsilon}{\varepsilon_0} - j\sigma \omega}}{\sin \psi + \sqrt{\frac{\varepsilon}{\varepsilon_0} - j\sigma \omega}} - \cos^2 \psi \]

\[ \sin \psi + \sqrt{(\varepsilon - j\sigma \omega)} - \cos^2 \psi \]
where
\[ x = \frac{5}{\omega \epsilon_0} \]
and
\[ \epsilon_r = \text{relative dielectric permittivity of the ground} \]

For vertical polarization

\[ 8_v = \frac{(\epsilon_r - jx) \sin \psi - \sqrt{(\epsilon_r - jx) - \omega^2 \epsilon_r}}{(\epsilon_r - jx) \sin \psi + \sqrt{(\epsilon_r - jx) - \omega^2 \epsilon_r}} \]

reflectivity coefficients are complex - a signal changes both magnitude and phase
reflectivity coefficients depend on the properties of the soil
reflectivity coefficients depend on the frequency of the radio wave

**Typical values of ground constants**

<table>
<thead>
<tr>
<th>Type of ground</th>
<th>( \delta (\text{in}) )</th>
<th>( \epsilon_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>poor ground</td>
<td>10^{-2}</td>
<td>4-7</td>
</tr>
<tr>
<td>average ground</td>
<td>5 \times 10^{-2}</td>
<td>15</td>
</tr>
<tr>
<td>wet ground</td>
<td>2 \times 10^{-2}</td>
<td>25-30</td>
</tr>
<tr>
<td>Sea water</td>
<td>5</td>
<td>81</td>
</tr>
<tr>
<td>Fresh water</td>
<td>1 \times 10^{-2}</td>
<td>81</td>
</tr>
</tbody>
</table>
Hand out graphs of reflecting coefficients for VHF/UHF.

Tabular frequencies:

- \( \nu_1 = 30 \text{ MHz} \)
- \( \nu_2 = 800 \text{ MHz} \)
- \( \nu_3 = 1900 \text{ MHz} \)
- \( \nu_4 = 2 \text{ GHz} \)

Observations:

1. For small incident angle,
   \[ \text{\textbf{E}}_x \approx 1 e^{j \psi} = -1 \]
   \[ \text{\textbf{E}}_y \approx 1 e^{j \psi} = -1 \]

2. Reflective coefficient for horizontal polarization changes slowly as a function of incident angle for wide range of angles.

3. Reflective coefficient for vertical polarization exhibits Brewster angle phenomena. Pseudo-Brewster angle occurs for \( \psi \approx 15^\circ \)

Generally \( \psi \) is small, \( \approx \) both horizontal and vertical polarizations have reflective waves approximately the same magnitude and opposite phase.
Propagahon over plane reflecting surface

The signal at the receiver consists of two components:

1) direct component
2) reflected component

That is

\[ E_T = E_d + S \cdot E_d \cdot \exp(-j\Delta \phi) \]

where \( S \) is the reflection coefficient and \( \Delta \phi \) is the phase difference that is attributed to difference in the length of propagation path.

Assuming that \( d \gg \lambda, \lambda = \psi \) is small. For small \( \psi \) the reflected component \( \approx -1 \)
Therefore,

\[ E_T = E_0 \left( 1 - \exp(-j\Delta \phi) \right) \]

\[ \Delta \Phi = \frac{\Delta d}{\lambda} \cdot 2\pi, \text{ where } \Delta d \text{ is difference in propagation path.} \]

From figure:

\[ \Delta d = \left[ \left( \frac{l_1 + l_2}{d} \right)^2 + d^2 \right]^{1/2} - \left[ \left( \frac{l_1 - l_2}{d} \right)^2 + d^2 \right]^{1/2} \]

\[ = d \left[ \left[ 1 + \left( \frac{l_1 + l_2}{d} \right)^2 \right]^{1/2} - \left[ 1 + \left( \frac{l_1 - l_2}{d} \right)^2 \right]^{1/2} \right] \]

\[ = d \left[ 1 + \frac{1}{2} \left( \frac{l_1 + l_2}{d} \right)^2 - 1 - \frac{1}{2} \left( \frac{l_1 - l_2}{d} \right)^2 \right] \]

\[ = \frac{d}{2} \cdot \frac{2l_1 l_2}{d^2} = \frac{2l_1 l_2}{d^2} \]

Therefore,

\[ \Delta \Phi = \frac{2l_1 l_2}{d^2} \cdot 2\pi \]

and

\[ E_T = E_0 \cdot \left( 1 - \exp(-j\frac{2\pi l_1 l_2}{d^2} \cdot 2\pi) \right) \]

If \( d \gg l_1 l_2 \), which is usually the case.
\[ E_t = \text{Fed} \left( 1 - 1 + 1 \frac{4\pi^2 L_2}{d^2} \right) = J \frac{4\pi L_2}{d^2} \cdot \text{Fed} \]

\[ W = \frac{1}{2} E_t^2 \varphi = \left( \frac{4\pi^2 L_2}{d^2} \right)^2 \left( \frac{E_t}{\varphi} \right)^2 \rightarrow Wd \]

\[ W = \left( \frac{4\pi^2 L_2}{d^2} \right)^2 \frac{P_t \cdot G_t}{4\pi d^2} \]

Finally,

\[ P_L = W \cdot \lambda \cdot c = W \cdot \frac{c^2}{4\pi} \cdot G_t = \frac{P_t \cdot G_t \cdot G_e \cdot c^2}{(4\pi d)^2} \cdot \left( \frac{4\pi L_2}{d^2} \right)^2 \]

or

\[ P_L = P_t \cdot G_t \cdot G_e \cdot \left( \frac{c \cdot L_2}{d^2} \right)^2 \]

Path loss \( P_L \), propagation over flat earth.

\[ P_L = \frac{P_t}{P_L} = \frac{d^4}{G_t G_e (L_1 L_2)^2} \sim d^4. \]

\[ P_L [\text{dB}] = 10 \log d - 20 \log L_1 - 20 \log L_2 - 5.7 - 10 \log G_e - 0.5 \log G_t. \]

Note:

1. The path loss increases 40 dB/dec as a function of distance.
2. There is no frequency dependence of the path loss—how so?
3. Path loss depends on the heights at TX & RX.
Example. Consider the situation depicted in Fig. x. Calculate the RSL of the mobile assuming the flat Earth 2-ray model.

\[ \theta_t = 30^\circ \]

\[ \theta_r = \theta_t = 30^\circ \]

\[ G_t = 12 \text{ dB} \]

\[ G_r = 0 \text{ dB} \]

\[ d = 7 \text{ km} \]

\[ \lambda = 2 \text{ m} \]

\[ P_L = 40 \log d - 20 \log \theta_t - 20 \log \lambda d - G_t - G_r = \]

\[ = 40 \log (7000) - 20 \log (30) - 20 \log (2) - 12 - 0 \]

\[ = 106.24 \text{ dB} \]

\[ RSL = ERP - P_L = 50 \text{ dBm} - 106.24 \text{ dB} = -56.24 \text{ dB} \]

Note: Two ray, flat Earth model usually underestimates the path loss.
Ground roughness.

Flat surfaces that have much larger dimension than the wavelength may be modeled as reflective surfaces. However, the roughness of such surfaces causes propagation effects that are different than pure specular reflection that we have discussed so far.

In the text, the author defines column:

\[ d_p > \frac{\lambda}{8 \sin \psi} \]

where
- \( \psi \) - incident angle
- \( \lambda \) - wavelength
- \( d_p \) - critical height

If \( d > d_p \) the surface is rough, otherwise the surface is smooth.

In practice, when known as irregular, \( d \) cannot be defined. For that reason, better way of treating roughness is standard deviation of surface height relative to its mean value.
\[ \delta \rho = \left[ E \left( \bar{\rho} - \bar{\rho}_0 \right)^2 \right]^{1/2} \]

Then, for rough surfaces the reflection coefficient needs to be modified through multiplication

\[ S_r = S_x \times S_s \]

where \( S_x \) is either horizontal or vertical reflection coefficient and \( S_s \) is given by

\[ S_s = \exp \left[ -8 \left( \frac{\pi \rho_s \sin \psi}{\lambda} \right)^2 \right] \]

1. As the ground about the reflection point is rough, the reflected wave becomes weaker. \( \Rightarrow \) Propagation becomes more like free space.

2. From practical standpoint, modeling ground roughness in VHF/UHF is not an easy task. Required information is not easy to gather.

3. In practice, existence of the roughness decreases slope of the path loss \( \Rightarrow \) slope is in mid 80's instead of 40-60 dB/dec.