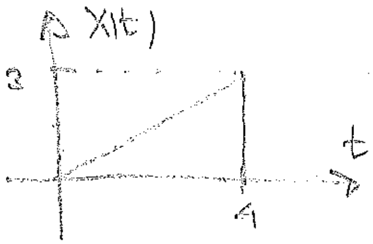
ECE 5201 - Linear Systems

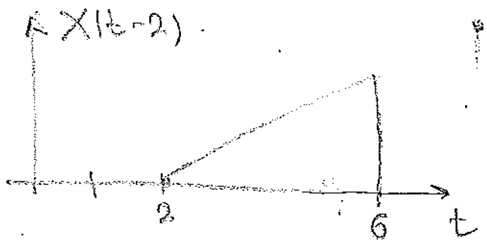
Review Session - 1

① A continuous time signal $x(t)$ is given in the figure. Sketch and label each of the following signals.

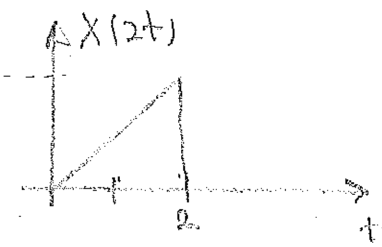
a) $x(t-2)$, b) $x(2t)$, c) $x(t/2)$, d) $x(-t)$



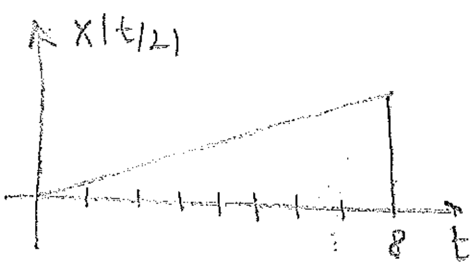
a) $x(t-2) \Rightarrow$ signal delayed by 2 units of time.



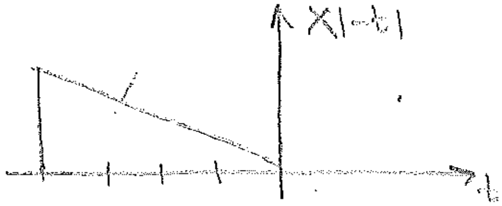
b) $x(2t) \rightarrow$ signal is 2 time faster



c) $x(t/2) \Rightarrow$ signal is two times slower



d) $X(-t) \Rightarrow$ signal is inverted on t-axis



(2) Find even and odd components of the signal $x(t) = \exp(jt)$

Method 1 $\quad X_e(t) = [X(t) + X(-t)]/2$

$$X_e(t) = \frac{e^{jt} + e^{-jt}}{2} = \cos t$$

$$X_o(t) = [X(t) - X(-t)]/2$$

$$X_o(t) = \frac{e^{jt} - e^{-jt}}{2j} \cdot j = j \sin(t)$$

Method 2 $\quad X(t) = \exp(jt) = \underbrace{\cos(t)}_{\text{even}} + \underbrace{j\sin(t)}_{\text{odd}}$

(3) Let $x_1(t)$ and $x_2(t)$ be periodic signals with periods T_1 and T_2 respectively. Under what condition is the sum $x(t) = x_1(t) + x_2(t)$ periodic, and what is the fundamental period of $x(t)$

$x_1(t)$ – periodic $\quad x_1(t + T_1) = x_1(t) \quad T_1$ – period of $x_1(t)$

$x_2(t)$ – periodic $\quad x_2(t + T_2) = x_2(t) \quad T_2$ – period of $x_2(t)$

$X(t) = x_1(t + mT_1) + x_2(t + nT_2) = X(t + T) \} \quad T$ – period of $X(t)$ $\quad (*)$

From (*), one obtains

$mT_1 = nT_2 = T \Rightarrow T_1/T_2 = m/n$

(4) Determine if the following signals are power signals, energy signals or neither

a) $x(t) = e^{-at}u(t),\ a>0$

$$\mathcal{E}_x = \int_{-\infty}^{+\infty} x^2(t)\,dt = \int_0^{+\infty} (e^{-at})^2\,dt = \int_0^{+\infty} e^{-2at}\,dt = \frac{1}{2a} \Rightarrow \text{energy signal}$$

b) $x(t) = A\cos(\omega_0 t + \theta)$

$$\mathcal{E}_x = \int_{-\infty}^{+\infty} A^2\cos^2(\omega_0 t + \theta)\,dt = \int_{-\infty}^{+\infty} \frac{A^2}{2}\left(1+\cos(2\omega_0 t + 2\theta)\right)dt \Rightarrow +\infty$$

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-\infty}^{+\infty} A^2\cos^2(\omega_0 t + \theta_0) = \frac{1}{T_0}\int_{T_0} \frac{A^2}{2}\left(1+\cos(2\omega_0 t+2\theta)\right)dt = \frac{A^2}{2}$$

This is a power signal

c) $x(t) = t\,u(t)$

$$\mathcal{E}_x = \int_{-\infty}^{+\infty} x^2(t) = \int_0^{+\infty} x^2\,dt \Rightarrow +\infty$$

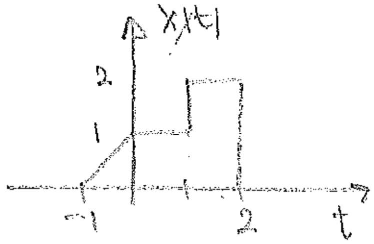
$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_T x^2(t)\,dt = \lim_{T\to\infty} \frac{1}{T}\int_0^T t^2\,dt = \lim_{T\to\infty} \frac{1}{T}\left.\frac{t^3}{3}\right|_0^T$$

$$= \lim_{T\to\infty} \frac{T^2}{3} \to +\infty$$

The signal is neither power nor energy

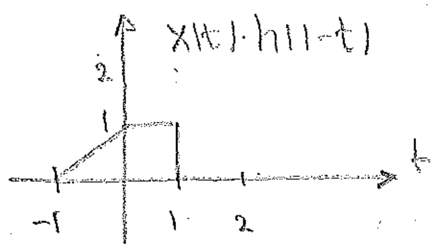
(5) A continuous signal $x(t)$ is presented in the figure. Sketch and label following signals

a) $x(t)\cdot u(1-t)$ b) $x(t)\cdot[u(t)-u(t-1)]$ c) $x(t)\cdot\delta(t-3/2)$



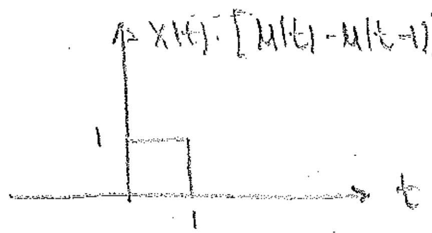
a) $x(t)\cdot u(1-t)$

$u(1-t)=\begin{cases}1 & t<1\\ 0 & \text{otherwise}\end{cases}$ $\quad x(t)=\begin{cases}x(t) & t\in(-1,2)\\ 0 & \text{otherwise}\end{cases}$



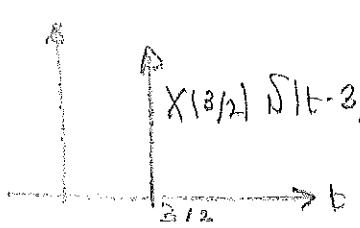
b) $x(t)\,[u(t)-u(t-1)]$

$u(t)-u(t-1)=\begin{cases}1 & t\in(0,1)\\ 0, & \text{otherwise}\end{cases}$



c) $x(t)\cdot\delta(t-3/2)$

$x(t)\cdot\delta(t-3/2)=x(3/2)\,\delta(t-3/2)$



(6) Consider a continuous time specified with differential equation

$(D^2+9)\,y(t) = (3D+2)\,x(t)$

a) Determine characteristic polynomial, characteristic equation, characteristic roots and characteristic modes of the system

b) Find $y_0(t)$, the zero-input component of the response $y(t)$ for $t \geq 0$, if the initial conditions are $y_0(0^-)=2$, $\dot{y}(0^-)=-1$

a) $Q(D) = D^2+9$

$Q(\lambda) = \lambda^2+9 \Rightarrow \lambda_{1/2} = \pm j3 \quad (\alpha=0,\ \beta=3)$

$y_1 = C_1 e^{j3t}, \quad y_2 = C_2 e^{-j3}$

b) $y_0(t) = A\cos(3t+\theta)$ - form of the zero input solution

$y_0(0) = A\cos(\theta) = 2$ (1)

$\dot{y}_0(t) = -3A\sin(3t+\theta) \quad \dot{y}_0(t) = -3A\sin(\theta) = -1$ (2)

From (1) & (2) $\quad -\dfrac{1}{3\tan(\theta)} = -2 \Rightarrow \theta = \operatorname{atan}(\frac{1}{6}) = 0.1651$

$A = \frac{2}{\cos(\theta)} = 2.0276$

Therefore: $y_0(t) = 2.0276\cos(3t+0.1651)$

(7) Find the impulse response of a system specified with differential equation

$(D^2+4D+3)\,y(t) = (D+5)\,x(t)$

<u>Step 1</u> $\quad Q(\lambda) = \lambda^2+4\lambda+3 = (\lambda+3)(\lambda+1) \Rightarrow \lambda_1 = -1,\ \lambda_2 = -3$

$y_n(t) = C_1 e^{-t} + C_2 e^{-3t}$

<u>Step 2</u> $y_n(0) = 0 \quad y_n^{(1)}(0) = 1$

$y_n(0) = C_1 + C_2 = 0$

$\dot{y}_n(0) = -C_1 - 3C_2 = 1 \qquad -2C_2 = 1 \Rightarrow C_2 = -1/2, \; C_1 = 1/2$

$y_n(t) = 1/2\left[e^{-t} - e^{-3t}\right]$

<u>Step 3.</u>

$h(t) = b_0 \delta(t) + [P(D)\, y_n(t)] \cdot u(t) \qquad (\text{since } M < N)$

$b_0 = 0, \qquad P(D) = D + 5$

$P(D)\, y_n(t) = (D+5) \cdot \left[\frac{1}{2} e^{-t} - e^{-3t}\right] =$

$= -1/2\, e^{-t} + 3e^{-3t} + \frac{5}{2} e^{-t} - 5e^{-3t} =$

$= 2e^{-t} - 2e^{-3t} = 2(e^{-t} - e^{-3t})$

$h(t) = 2(e^{-t} - e^{-3t}) \cdot u(t) \qquad$ (we're sure h(t) is valid only for $t \geq 0$)

(8) Consider a system with impulse response

$h(t) = 4e^{-2t} \cos(3t) \cdot u(t)$

Determine the output of the system to the inputs

a) $x(t) = u(t)$

b) $x(t) = e^{-t}\, u(t)$

a) $y(t) = x(t) * h(t) = \int_{\tau=-\infty}^{+\infty} x(\tau) h(t-\tau) d\tau$

In this case.

$$y(t) = \int_{\tau=-\infty}^{+\infty} 4 e^{-2\tau} \cos(3\tau) u(\tau) \cdot u(t-\tau) \, d\tau$$

$$= \int_{\tau=0}^{t} 4 e^{-2\tau} \cos(3\tau) \, d\tau$$

Table integral: $\int e^{bx} \cos(ax) \, dx = \frac{1}{a^2+b^2} e^{bx} (a \sin ax + b \cos ax)$

Therefore.

$$y(t) = 4 \cdot \frac{e^{-2\tau}}{3^2+2^2} \cdot \left[3 \sin(3\tau) + (-2) \cdot \cos(3\tau) \right] \Big|_0^t$$

$$y(t) = \frac{4}{13} \left\{ e^{-2t} \left[3 \sin(3t) - 2 \cos(3t) \right] - 1 \cdot \left[0 - 2 \right] \right\} u(t)$$

$$= \frac{4}{13} \left[e^{-2t} \left[3 \sin(3t) - 2 \cos(3t) \right] + 2 \right] \cdot u(t).$$

b) $y(t) = \int_{\tau=-\infty}^{+\infty} 4 e^{-2\tau} \cos(3\tau) u(\tau) \cdot e^{-(t-\tau)} u(t-\tau) \, d\tau =$

$$= 4 e^{-t} \int_{\tau=0}^{t} e^{-\tau} \cos(3\tau) \, d\tau =$$

$$= 4 e^{-t} \frac{e^{-\tau}}{1^2+3^2} \cdot \left[3 \sin(3\tau) - \cos(3\tau) \right] \Big|_0^t$$

$$= 4 e^{-t} \left(\frac{e^{-t}}{10} \left[3 \sin(3t) - \cos(3t) \right] - \frac{1}{10} \left[-1 \right] \right)$$

$y(t) = \frac{4}{19}\left[e^{-2t}\left(3\sin(3t) - \cos(3t)\right) + 1\right] \cdot u(t)$

) Determine output of the system given by.

$(D^2 + 6D + 25)\,y(t) = (D+3)\,x(t)$

$y(0^+) = 0, \quad \dot{y}(0^+) = 2, \quad x(t) = u(t)$

Zero input response.

$Q(\lambda) = \lambda^2 + 6\lambda + 25, \quad \lambda_{1/2} = -3 \pm j4$

$y_0(t) = A e^{-3t} \cos(4t + \theta)$

$y_0(0) = A\cos(\theta) = 0 \qquad \Rightarrow \theta = \pi/2$

$\dot{y}_0(t) = -3A\cos(4t+\theta) - 4Ae^{-3t}\sin(4t+\theta)$

$\dot{y}_0(0) = -3A\cos(\theta) - 4A\sin(\theta) = 2 \qquad -4A = 2 \Rightarrow A = -1/2$

$y_0(t) = -1/2\, e^{-3t}\cos(4t + \pi/2) = 1/2\, e^{-3t}\sin(4t) \quad u(t)$

) Impulse response.

a) $y_n(t) = A e^{-3t}\cos(4t + \theta)$

$y_n(0) = 0, \; y_n^{(1)}(0) = 1$

$y_n(0) = A\cos(\theta) = 0 \qquad \rightarrow \theta = \pi/2$

$\dot{y}_n(0) = -3A\cos(\theta) - 4A\sin(\theta) = 1 \qquad -4A = 1 \Rightarrow A = -1/4$

$y_n(t) = 1/4\, e^{-3t}\sin(4t)$

b) $h(t) = b_0 \delta(t) + (P(D) \cdot y_n(t)) \cdot u(t)$

; $b_0 = 0$,

$P(D) \cdot y_n(t) = (D+3)[\frac{1}{4} e^{-3t} \sin(4t)] =$

$= -\frac{3}{4} e^{-3t} \sin(4t) + \frac{1}{4} \cdot 4 e^{-3t} \cos(4t) + \frac{3}{4} e^{-3t} \sin(4t)$

$= e^{-3t} \cos(4t)$

$h(t) = e^{-3t} \cos(4t) \cdot u(t)$ – impulse response.

iii) Zero state response

$y_{zs}(t) = x(t) * h(t) = \int_{\tau=-\infty}^{+\infty} h(\tau) u(t-\tau)\, d\tau =$

$= \int_{\tau=-\infty}^{+\infty} e^{-3\tau} \cos(4\tau)\, u(\tau) \cdot u(t-\tau) = \int_{\tau=0}^{t} e^{-3\tau} \cos(4\tau)\, d\tau$

$= \frac{1}{3^2+4^2} e^{-3\tau} [4\sin(4\tau) - 3\cos(4\tau)] \Big|_0^t =$

$= \frac{1}{25} \Big[e^{-3t} [4\sin(4t) - 3\cos(4t)] + 3 \Big]$

iv) total response

$y_T(t) = \{\frac{1}{2} e^{-3t} \cdot \sin(4t) + \frac{1}{25} [e^{-3t} (4\sin(4t) - 3\cos(4t)) + 3\} u(t)$.