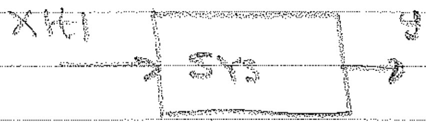
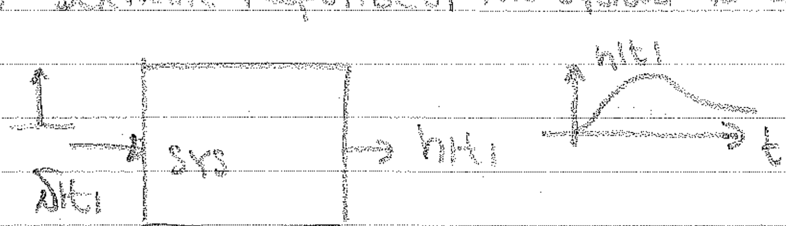
Superposition and linear systems

Linear systems — systems where superposition applies



Two principle approaches in system analysis

1) Determine response of the system to an impulse

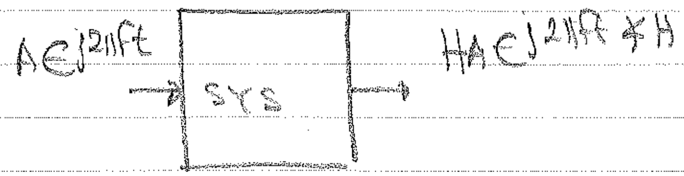


$$x(t) = \int_{\tau=-\infty}^{+\infty} x(\tau)\cdot\delta(t-\tau)$$ - Signal is decomposed as a sum of delta pulses

$$y(t) = \int_{\tau=-\infty}^{+\infty} x(\tau)\cdot h(t-\tau)$$ - output is decomposed as a sum of responses to individual pulses.

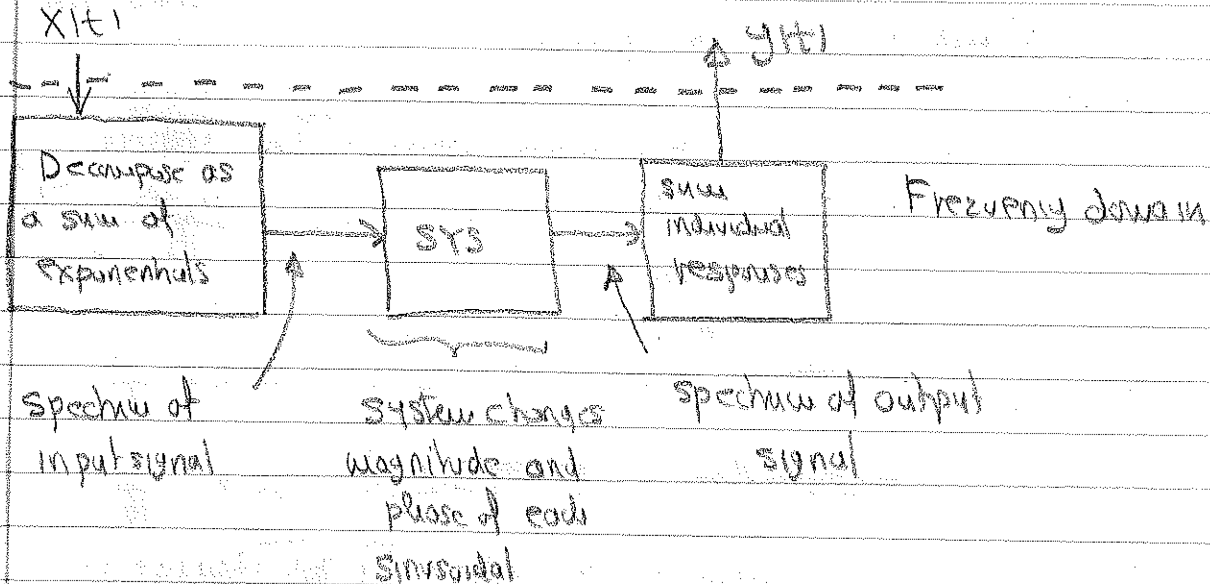
This approach is basis of the analysis in time domain

2) Determine response of the system to an exponential



$f \in (0, +\infty)$

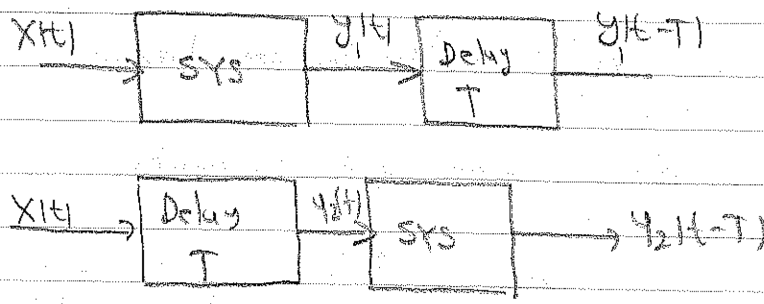
* linear system changes only magnitude and the phase of the exponential - It cannot change its frequency.



* The above outlined approach is basis for the frequency domain analysis.
* In everyday practice we use both time domain and frequency domain analysis → the two are of equal importance.

<u>Time invariant and time variant systems.</u>

Time invariant systems - parameters do not change over time



If $y_1(t-T) = y_2(t-T)$ are the same the system is time invariant

nder Digital, discrete time systems are time invariant.

note2: Analog systems made of RLC components and active circuits are assumed as time invariant

note3: Example of time variant systems are various wireless channels
1) mobile terrestrial channel
2) satellite channel
3) underwater communication channel

Instantaneous and Dynamic Systems.

In general - system's output at time t depends on the entire past of the system - DYNAMIC SYSTEM (with memory)
In special case - system's output at time t depends only on the input at time t - INSTANTANEOUS SYSTEMS (memoryless)

Example1. Circuits with capacitors or inductors are dynamic systems

Example 2. Circuits with just resistors are memoryless systems

Note: Memoryless systems are special case of dynamic systems with zero memory.

Causal and Non causal systems

Causal system - A system for which output at time t_0 depends on inputs from times $t \leq t_0$.

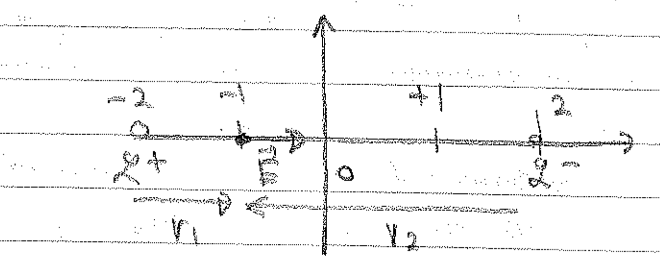
Non causal system - the output of the system depends on past and future values of input

note 1: All physical systems that are operating in "real" time are causal systems.

Non causal systems appear in two important scenarios

Scenario 1. Independent variable is not time.

Example. Consider electric field at point $x=-1$ in the figure



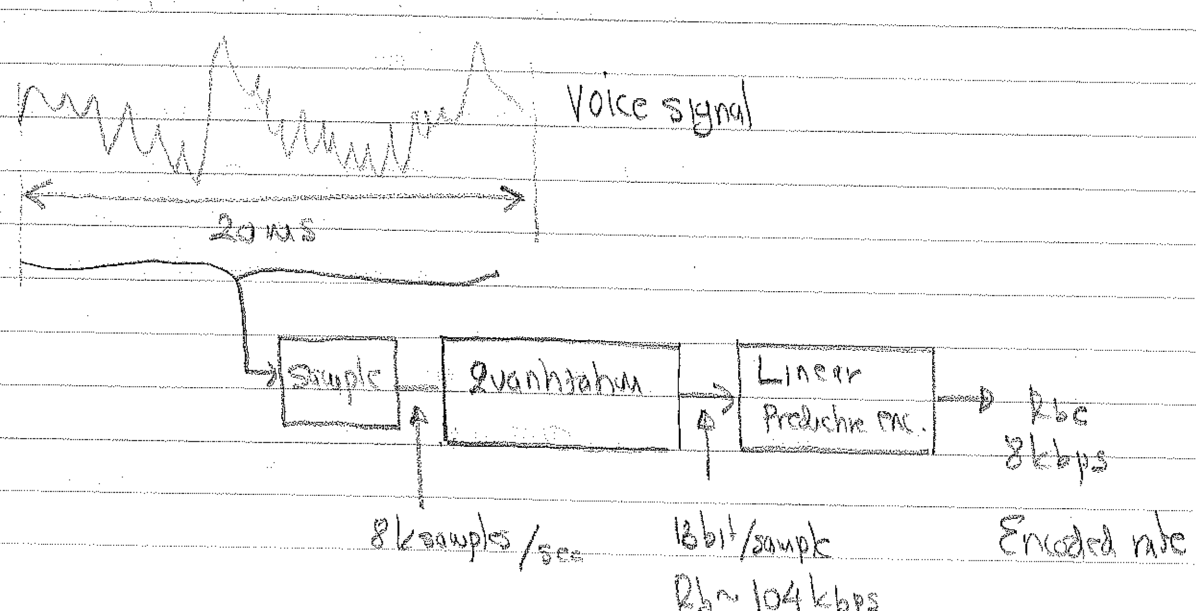
$$\vec{E} = \left(\frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} \right) \vec{e}_x$$

The "response" of the system at $x=-1$ depends on the input that occurs at $x=2$.

Scenario 2. The processing of the signal is not real time.

In many practical situation the signal is prerecorded before it is processed. Therefore, the system may have access to the entire (or a part) of the signal before it processes its very begining. It turns out that this one of the most effective approaches in signal processing

Example. Voice processing in cellular telephony.



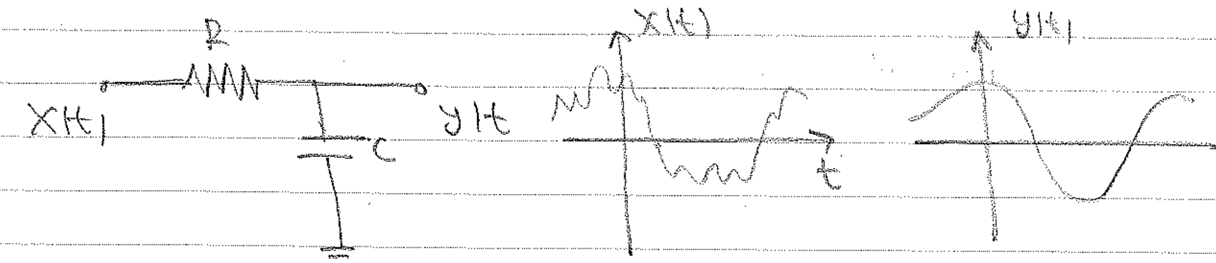
Note 1. Non-causal systems become physically realizable if enough delay is introduced into the system

Continuous time and Discrete time systems

Systems whose inputs and outputs are continuous signals are referred to as continuous systems.

Systems whose inputs and outputs are discrete in time are referred to as the discrete-time systems

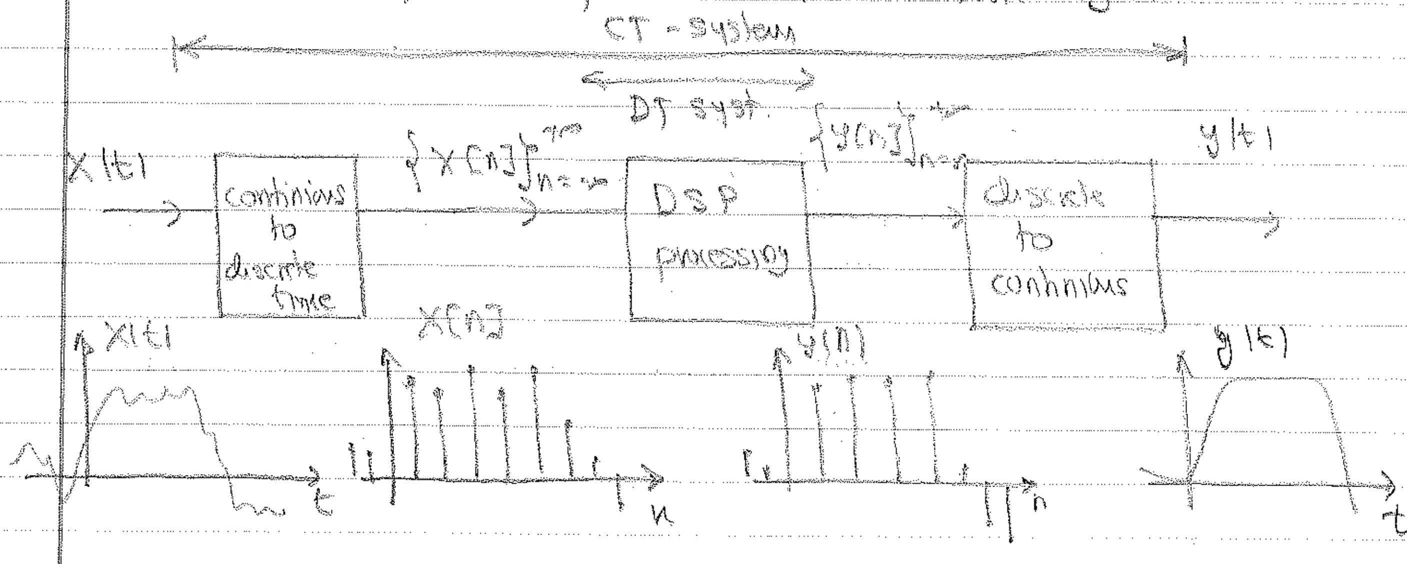
Example: Analog RLC filter - continuous system



Rc - lowpass filter   input   output

Example DSP hardware implements discrete time systems.

Note: In modern signal processing systems it is common to use discrete-time systems to process continuous time signals

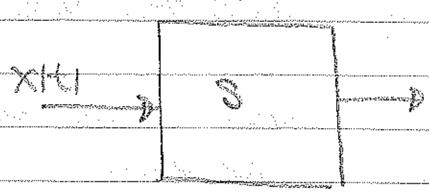


Key part of the system is conversions between continuous time and discrete time. Mathematically correct way to perform this conversion is explained by Sampling Theorem.

Analog and Digital Systems

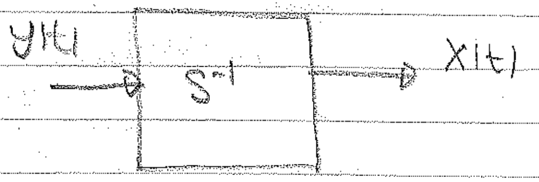
* Systems whose inputs and outputs are analog and who perform signal processing in analog domain are referred to as the analog systems.
* Systems whose inputs and outputs are digital are referred to as digital systems.

Invertible and Non-invertible Systems

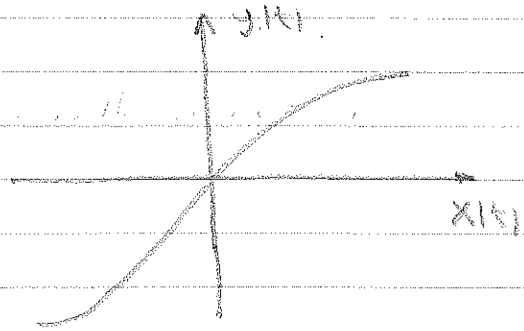


$S: x(t) \rightarrow y(t)$

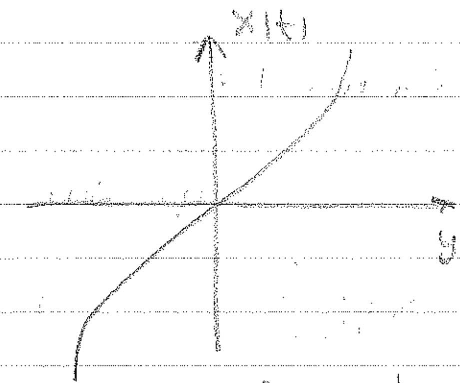
System S performs mapping of x(t) into y(t). If there exists a system S^{-1} that performs a mapping y(t) back to x(t) the system is set to be invertible. For invertible systems there is 1-1 mapping between input and output.



Example: Invertible system - nonlinear amplifier with no memory.

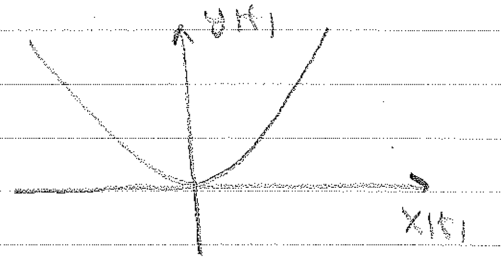


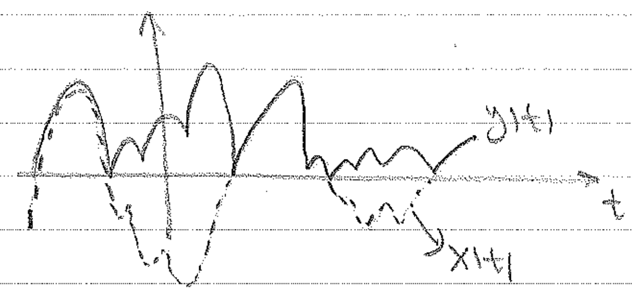
mapping for S



mapping for S^{-1}

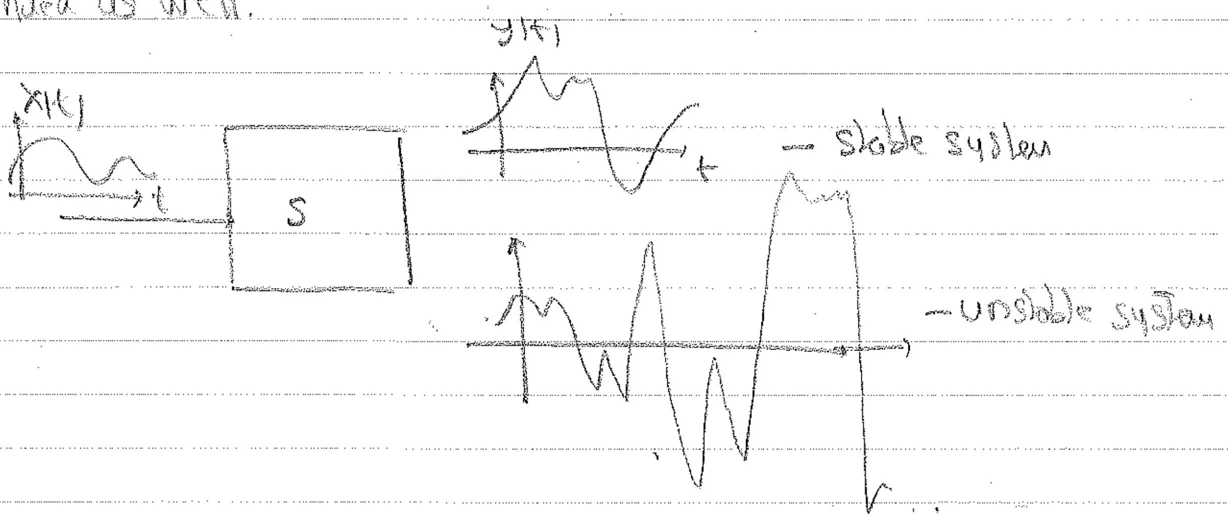
Example 2 Non-invertible system - rectifier





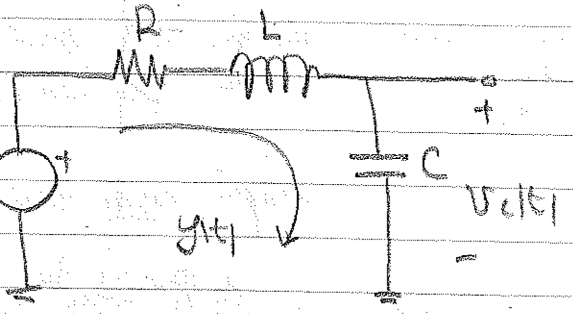
Stable and Non-stable systems. (BIBO stability)

System is stable if for any bounded inputs the outputs are bounded as well.



note: Unstable systems usually have some sort of positive feedback that feeds some of system's outputs back to the input.

Example 1.10



Find the relationship

$S: x(t) \rightarrow y(t)$

$R = 3\,\Omega$
$C = 1/2\,F$
$L = 1\,H$

$$y(t) = C\frac{dv_c(t)}{dt} \quad (1)$$

$$v_c(t) = x(t) - R\cdot y(t) - L\frac{dy(t)}{dt} \quad (2)$$

$$y(t) = C\cdot\frac{d}{dt}\left[x(t) - Ry(t) - L\frac{dy(t)}{dt}\right]$$

$$LC\frac{d^2y(t)}{dt^2} + RC\frac{dy(t)}{dt} + y(t) = C\frac{dx(t)}{dt}$$

$$\frac{1}{2}\frac{d^2y(t)}{dt^2} + 3\cdot\frac{1}{2}\frac{dy(t)}{dt} + y(t) = \frac{1}{2}\frac{dx(t)}{dt}$$

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt}$$

System in figure is a linear, time invariant, causal, analog, continuous time and stable.

Sample problems.

17-1 17-7
17-2 17-8
17-4 Review MATLAB Session 1. Working with functions.