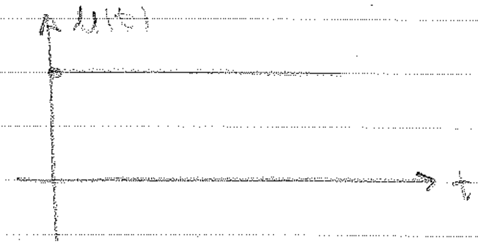
ECE 5201 Linear Systems — Lecture 2.

Some useful signals

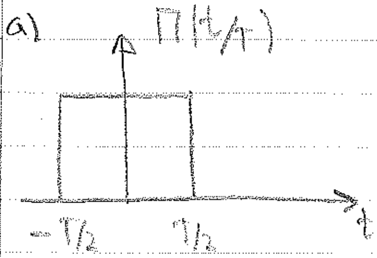
1) Unit step function - "switch-on" function

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



Examples of use:

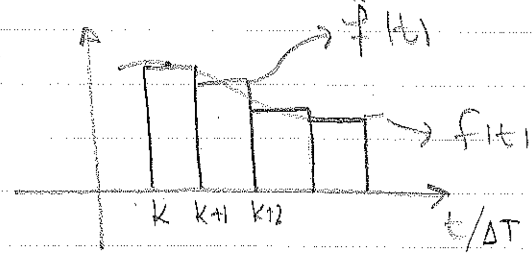
a)



$$\Pi(t/T) = u(t+T/2) - u(t-T/2)$$

$\Pi(t/T)$ - pulse centered on the origin and having width of T.

b)



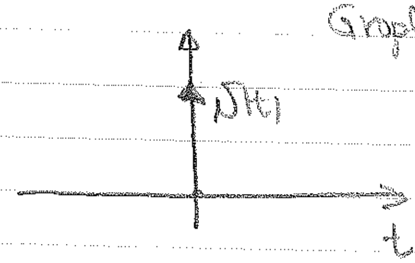
$$\hat{f}(t) = \sum_{k=-\infty}^{+\infty} \frac{f(k\Delta T) + f((k+1)\Delta T)}{2} \, u(t-k\Delta T) - u(t-(k+1)\Delta T)$$

as $\Delta T \to 0$ $\hat{f}(t) \to f(t)$

note: every function can be approximated using the unit step functions.

2) The unit impulse function $\delta(t)$

Graphical representation

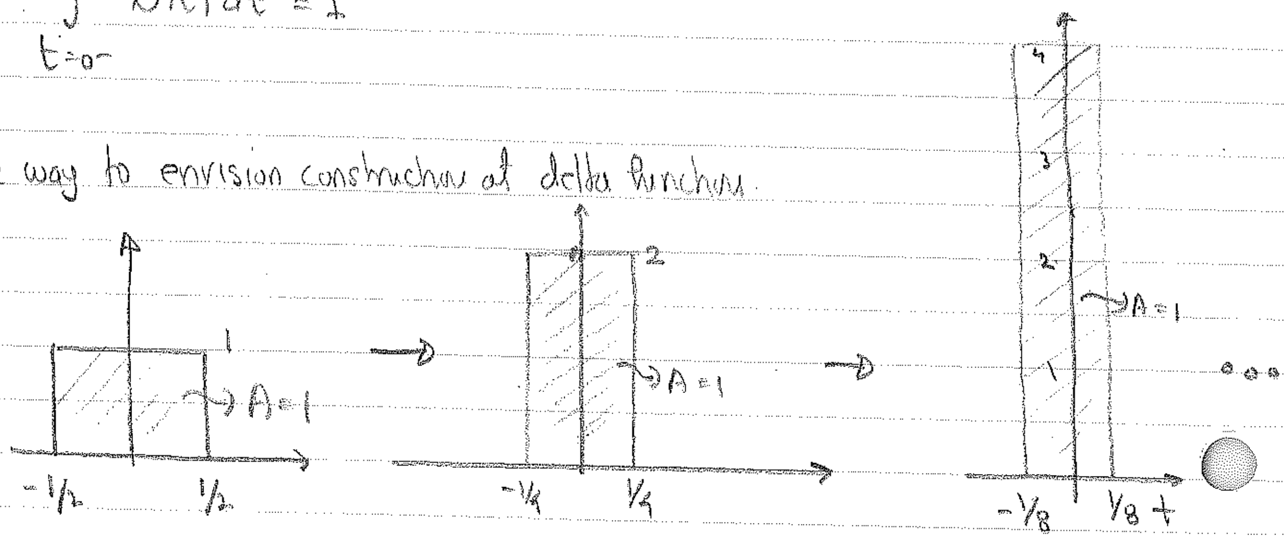


1° $\delta(t) = 0, \ \forall t \neq 0$

2° $\delta(t) \to \infty, \ t = 0$

3° $\displaystyle\int_{t=0^-}^{0^+} \delta(t)\, dt = 1$

One way to envision construction of delta function.



As the process continues, the function becomes "higher" and "narrower" and in the limit it approaches infinity

a) Sampling (sifting) property of $\delta(t)$

$$\int_{t=-\infty}^{+\infty} f(t)\cdot\delta(t-T)\, dt = f(t)$$

Proof:
$$\int_{t=-\infty}^{+\infty} f(t)\cdot\delta(t-T)\,dt = \int_{t=T_-}^{T_+} f(t)\,\delta(t-T)\,dt =$$

$$= f(T)\cdot\int_{t=T_-}^{T_+} \delta(t-T)\,dt = f(T)\int_{\nu=0^-}^{0^+} \delta(\nu)\,d\nu = f(T)$$

Using Sifting property of $\delta(t)$ function

$$f(t) = \int_{\tau=-\infty}^{+\infty} f(\tau)\,\delta(t-\tau)\,d\tau = \int_{\tau=-\infty}^{+\infty} f(t-\tau)\cdot\delta(\tau)\,d\tau$$

3) Exponential function

$f(t) = \exp(st)$, $s = \sigma + j\omega$ - complex frequency

$f(t) = \exp((\sigma + j\omega)t) = \exp(\sigma t)\cdot\exp(j\omega t)$

$= \exp(\sigma t)\cdot[\cos\omega t + j\sin\omega t]$ (*)

Where in (*), the use was made of the Euler's identity

$\exp[jx] = \cos(x) + j\sin(x)$

__Note:__ Exponential is used to define sin and cos.

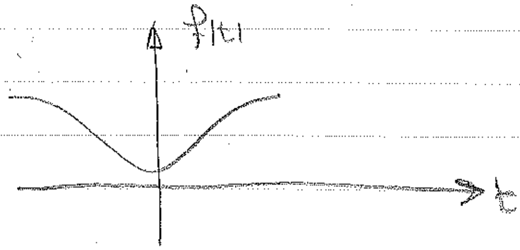
$$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$$

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$$

Remember: $e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

Even and Odd functions

1) $f(t) \Rightarrow$ even function if $f(t) = f(-t)$



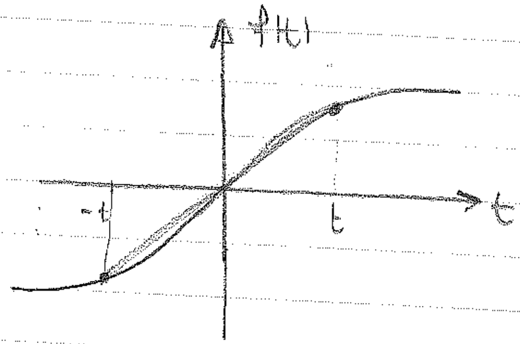
Even functions have symmetry about y axis, i.e. y axis may be seen as a "mirror" for the "left" and "right" part of the function

Examples of even functions:

$f(t) = \text{const}$
$f(t) = 2t^2$
$f(t) = \cos(2\pi f t) + t^4$

2) $f(t)$ - odd function

$f(t) \to$ odd if $f(t) = -f(-t)$



Odd functions have symmetry about the origin

Example of odd functions.

$f(t) = 3t - 2t^3$
$f(t) = \sin(2\pi f t)$, $f \neq 0$
$f(t) = \operatorname{atan}(bt)$, $b \neq 0$

Note1: A function may be even, odd or neither

note 2: odd x odd $\to$ even
odd x even $\to$ odd
even x even $\to$ even

note 3: if $f(t)$ - odd $\int_{-T}^{+T} f(t)\,dt = 0$

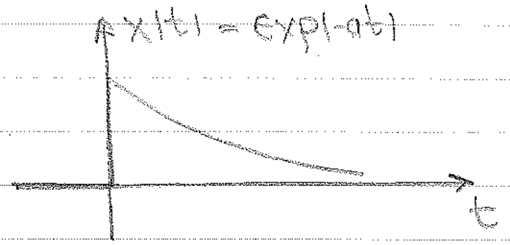
if $f(t)$ - even $\int_{-T}^{T} f(t)\,dt = 2\int_{0}^{T} f(t)\,dt$

Every signal may be represented as a sum of its even part and its odd part. To demonstrate, consider an arbitrary signal $x(t)$

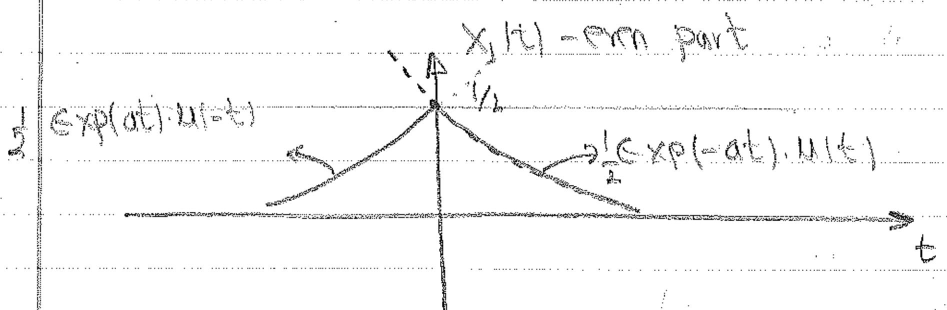
$$x(t) = \frac{1}{2}\underbrace{\left[x(t) + x(-t)\right]}_{\text{even part}} + \frac{1}{2}\underbrace{\left[x(t) - x(-t)\right]}_{\text{odd part}}$$

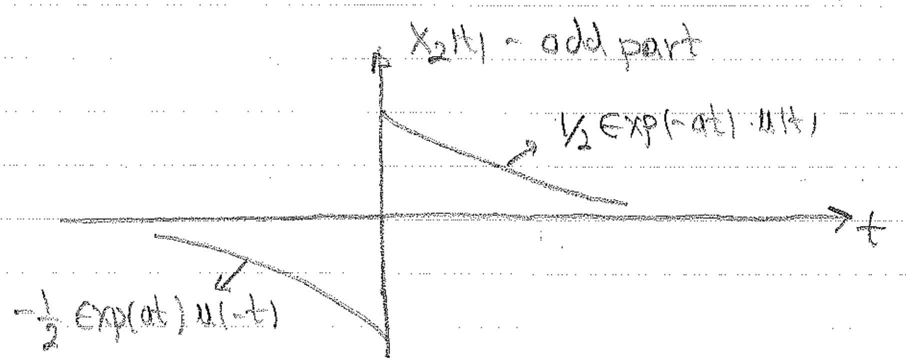
Example. Consider a function

$$x(t) = \exp(-at)\cdot u(t), \quad a > 0$$



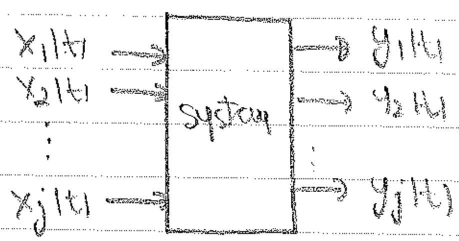
$$x(t) = \frac{1}{2}\left[\exp(-at)\,u(t) + \exp(at)\,u(-t)\right] + \frac{1}{2}\left[\exp(-at)\,u(t) - \exp(at)\,u(-t)\right] = x_1(t) + x_2(t)$$





Systems

"A system is an entity that processes a set of signals (inputs) and produces another set of signals (outputs)" - book definition



System theory allows

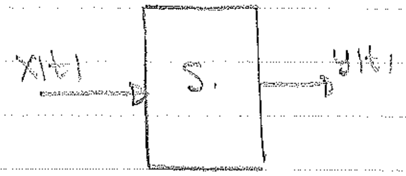
1) description of the system
2) calculation of outputs for given set of inputs and known system's state
3) design of systems that provide desired mapping between inputs and outputs

Classification of systems

1) Linear and nonlinear systems
2) constant parameter / time varying parameters
3) memoryless or with memory
4) causal and noncausal
5) continuous time and discrete time
6) analog and digital
7) invertible and noninvertable
8) stable and unstable
9) deterministic and probabilistic

Linear and Nonlinear systems

Linear systems are systems that support superposition.



Consider a system S. which maps $x(t) \to y(t)$

$$\text{If} \quad x_1(t) \xrightarrow{S} y_1(t)$$
$$x_2(t) \xrightarrow{S} y_2(t)$$

$$\text{and} \quad A \cdot x_1(t) + B \cdot x_2(t) \xrightarrow{S} A \cdot y_1(t) + B \cdot y_2(t)$$

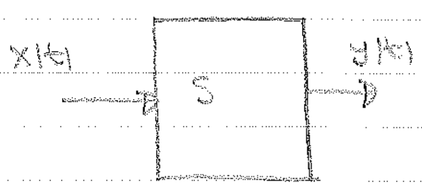
the system is considered as linear system

__note1:__ In practical application very few systems are truly linear across all possible $x_1(t)$ and $x_2(t)$. However, for some limited classes (ranges) of inputs many systems are designed to behave as linear systems.

__note2:__ Linear systems are typically described using linear differential or difference equations

__Example__ Consider a system described by following differential equation

$$\frac{dy(t)}{dt} + 3y(t) = x(t)$$



$$\text{Let} \quad x_1(t) \xrightarrow{S} y_1(t)$$
$$x_2(t) \xrightarrow{S} y_2(t)$$

and let $x(t) = k_1 x_1(t) + k_2 x_2(t)$

One obtains

$$\frac{dy_1(t)}{dt} + 3y_1(t) = x_1(t) \quad / \cdot k_1$$

$$\frac{dy_2(t)}{dt} + 3y_2(t) = x_2(t) \quad / \cdot k_2 \quad +$$

$$\frac{d}{dt}\left(k_1 y_1(t) + k_2 y_2(t)\right) + k_1 3y_1(t) + k_2 3y_2(t) = k_1 x_1(t) + k_2 x_2(t)$$

$$\frac{d}{dt}\underbrace{\left(k_1 y_1(t) + k_2 y_2(t)\right)}_{y(t)} + 3\underbrace{\left(k_1 y_1(t) + k_2 y_2(t)\right)}_{y(t)} = \underbrace{k_1 x_1(t) + k_2 x_2(t)}_{x(t)}$$

Therefore, when the input is $x(t) = k_1 x_1(t) + k_2 x_2(t)$, the output is $y(t) = k_1 y_1(t) + k_2 y_2(t)$

Suggested problems:

1-4-1	1-5-1	1-5-12
1-4-3	1-5-3	
1-4-4	1-5-6	
1-4-6	1-5-8	
1-4-10	1-5-11	