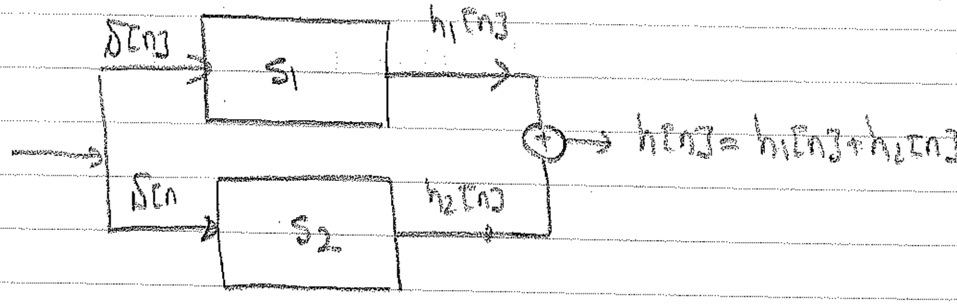
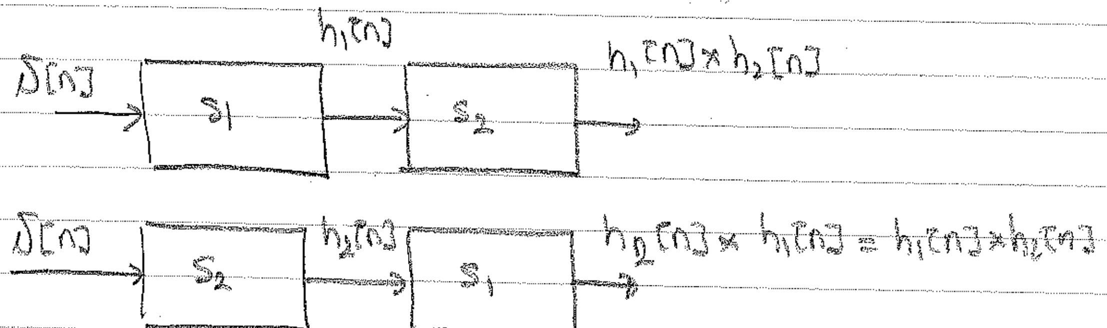
Interconnecting systems

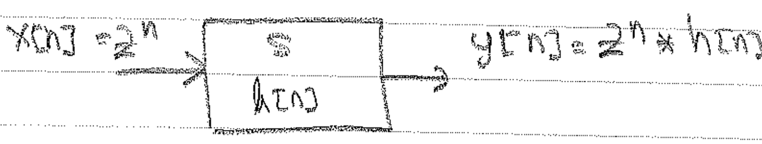
1) parallel connection



2) Series connection



Response of DT system to everlasting exponential



z^n - exponential, in general case $z = a + jb = re^{j\theta}$ - complex number

$$y[n] = z^n * h[n] = \sum_{m=-\infty}^{+\infty} h[m] \cdot z^{n-m} = z^n \cdot \sum_{m=-\infty}^{+\infty} h[m] \cdot z^{-m}$$

$= z^n \cdot H(z)$, $H(z)$ - referred to as the transfer function

Another way to define $H(z)$ is

$$H(z) = \left.\frac{\text{output signal}}{\text{input signal}}\right|_{\text{zero initial conditions},\ x[n]=z^n - \text{input}}$$

Consider

$$Q(E)\cdot y[n] = P(E)\cdot x[n]$$

Let $x[n] = z^n$; then.

$$Q(E)\cdot y[n] = Q(E)\cdot\{H(z)\cdot z^n\} = H(z)\cdot Q(z)\, z^n$$
$$P(E)\cdot x[n] = P(z)\, z^n$$

Therefore.

$$H(z)\cdot Q(z)\cdot z^n = P(z)\cdot z^n, \text{ or}$$

$$H(z) = P(z)/Q(z)$$ – obtaining of the transfer function directly from the difference equation of the system

<u>Total response of LTID system</u>

$$x[n] \longrightarrow \boxed{\begin{matrix} S. \\ h[n],\ IC \end{matrix}} \longrightarrow y[n]$$

$$y[n] = y_{zi}[n] + y_{zs}[n] = \underbrace{\sum_{j=1}^{N} c_j \gamma_j^{\,n}}_{\text{natural modes.}} + \underbrace{x[n] * h[n]}_{\text{convolution of the input and impulse response of the system.}}$$

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Example. Consider system

$$y[n+2] - 0.6\,y[n+1] - 0.16\,y[n] = 5x[n+2] \quad (*)$$

initial conditions: $y[-1] = 0$, $y[-2] = 25/4$

input: $x[n] = 4^{-n}u[n]$

Step1. Zero input response

$$Q(\gamma) = \gamma^2 - 0.6\gamma - 0.16$$

$$\gamma_{1/2} = \frac{0.6 \pm \sqrt{0.36 + 0.64}}{2} = \frac{0.6 \pm \sqrt{1.00}}{2}$$

$\gamma_1 = -0.2 \quad \gamma_2 = 0.8$

$$y_{zi}[n] = c_1(-0.2)^n + c_2(0.8)^n$$

Step2. Determine impulse response.

$$(E^2 - 0.6E - 0.16)\,y[n] = 5E^2\,x[n]$$

$$h[n] = \frac{b_N}{a_N}\delta[n] + y_c[n]\cdot u(n),$$

$h[n] = y_c[n] = k_1(-0.2)^n + k_2(0.8)^n$, since $b_N = 0$ (*)

To determine k_1 and k_2, assume $x[n] = \delta[n]$, From (*)

one has: $y[n+2] = 0.6\,y[n+1] + 0.16\,y[n] + 5\,x[n+2]$ or

$$y[n] = 0.6\,y[n-1] + 0.16\,y[n-2] + 5\,x[n]$$

therefore $\quad h[n] = 0.6\,h[n-1] + 0.16\,h[n-2] + 5\,\delta[n]$

$h[0] = 5.$

$h[1] = 0.6h[0] + 0.16h[-1] + 5\delta[1] = 0.6 \times 5 = 0.3.$

From (*) $h[0] = k_1 + k_2 = 5$, $h[1] = -0.2k_1 + 0.8k_2 = 3$ $\Big\}$ $k_1 = 1$, $k_2 = 4$

Therefore $h[n] = [(-0.2)^n + 4.(0.8)^n]u[n].$

Step 3. Zero state response

$$y_{zs}[n] = x[n] * h[n] = \sum_{m=-\infty}^{+\infty} x[m]h[n-m]$$

$$= \sum_{m=-\infty}^{+\infty} 4^{-m}u[m] \cdot [(-0.2)^{n-m} + 4(0.8)^{n-m}] \cdot u[n-m]$$

$$= \sum_{m=0}^{n} 4^{-m}[(-0.2)^{n-m} + 4(0.8)^{n-m}] =$$

$$= \sum_{m=0}^{n} (-0.2)^n \cdot [4 \times (-0.2)]^{-m} + \sum_{m=0}^{n} (0.8)^n \cdot (4 \times 0.8)^{-m} =$$

$$= (-0.2)^n \sum_{m=0}^{n} (-0.8)^{-m} + (0.8)^n \sum_{m=0}^{n} (3.2)^{-m}$$

$$= (-0.2)^n \sum_{m=0}^{n} \left(-\frac{1}{0.8}\right)^m + (0.8)^n \cdot \sum_{m=0}^{n} \left(\frac{1}{3.2}\right)^m$$

$$= (-0.2)^n \frac{1-(-1/0.8)^{n+1}}{1-(-1/0.8)} + (0.8)^n \frac{1-(1/3.2)^{n+1}}{1-1/3.2}$$

$$= \left(-1.26(4)^{-n} + 0.444(-0.2)^n + 5.81(0.8)^n\right)u[n]$$

Step 4. Total response

$$y_{TOT}[n] = y_{zi}[n] + y_{zs}[n] =$$

$$= C_1(-0.2)^n + C_2(0.8)^n + [-1.26\, 4^{-n} + 0.444.(-0.2)^n + 5.81 \times (0.8)^n]u[n]$$

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Using initial conditions

$y_{TOT}[-1] = C_1 \cdot (-0.2)^{-1} + C_2 (0.8)^{-1} = 0$

$y_{TOT}[0] = C_1 + C_2 + (-1.26 + 0.444 + 5.81) = 25/4$

solving for C_1 & C_2 => $C_1 = 0.2$ and $C_2 = 0.8$

Final answer:

$$y_{TOT}[n] = 0.2 \times (0.2)^n + 0.8\,(0.8)^n$$

$$-1.26\,(4)^{-n} + 0.444\,(-0.2)^n + 5.81 \cdot 0.8^n, \quad n \geq 0$$

Summary.

Step 1: Determine zero input response (leave the constants)

Step 2: Determine impulse response of the system

Step 3: Determine zero state response as a convolution of input signal and impulse response

Step 4: Determine total response and use initial conditions to determine the values of the constants.

Problems:

3.8-10 3.8-28
3.8.-11 3.9.-2 use 4 step approach
3.8-12 3.9-3 use 4 step approach
3.8-13 3.9-4 use 4 step approach
3.9.-8 use 4 step approach