ECE 5201 - Linear Systems   Lecture 1

* Review Class outline

Signal — function of time variable

$x(t)$ — signal; may be continuous, discrete, analog, digital, real, complex, ...
; may be voltage, current, numbers, ...

Signal Energy

$$\mathcal{E}_x = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt$$ — area under the signal curve

1) $x(t)$ — real $x(t) = x_r(t)$

$$\mathcal{E}_x = \int_{-\infty}^{+\infty} |x_r(t)|^2 \, dt = \int_{-\infty}^{+\infty} x_r^2(t) \, dt$$

2) $x(t)$ — complex $x(t) = x_r(t) + j\, x_i(t)$

$$\mathcal{E}_x = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt = \int_{-\infty}^{+\infty} |x_r(t) + j\, x_i(t)|^2 \, dt$$

$$= \int_{-\infty}^{+\infty} \left(x_r^2(t) + x_i^2(t) \right) dt =$$

$$= \int_{-\infty}^{+\infty} x_r^2(t) \, dt + \int_{-\infty}^{+\infty} x_i^2(t) \, dt = \mathcal{E}_{xr} + \mathcal{E}_{xi}$$

note: Signals that have finite signal energy are called energy signals

Signal Power:

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_T |x(t)|^2 \, dt$$

Power is average energy per time

1) $x(t)$ - real $\quad x(t) = x_r(t)$

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_T |x(t)|^2 dt = \lim_{T \to \infty} \frac{1}{T} \int_T x^2(t)\, dt$$

2) $x(t)$ - complex $\quad x(t) = x_r(t) + j x_i(t)$

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_T |x(t)|^2 dt = \lim_{T \to \infty} \frac{1}{T} \int_T |x_r(t) + j x_i(t)|^2 dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_T \left(x_r^2(t) + x_i^2(t) \right) dt = P_{x_r} + P_{x_i}$$

note: For complex signals:

+ total energy is sum of the energies in real and imag parts
+ total power is sum of the powers in real and imag parts

Example 1.2 Determine the power and rms value of following signals

a) $x(t) = C \cos(\omega_0 t + \theta)$, $\omega_0 \neq 0$

b) $x(t) = C_1 \cos(\omega_1 t + \theta_1) + C_2 \cos(\omega_2 t + \theta_2)$, $\omega_1 \neq \omega_2 \neq 0$

c) $x(t) = D e^{j\omega_0 t}$

a) $$P_x = \lim_{T \to \infty} \frac{1}{T} \int_T \left[C \cos(\omega_0 t + \theta) \right]^2 dt$$

* Power is average energy per unit time
* for periodic signal averageing can be done over one period

$$P_x = \frac{1}{T} \int_T C^2 \cos^2(\omega_0 t + \theta)\, dt, \quad T = \frac{2\pi}{\omega_0}$$

$$P_x = \frac{1}{T} \int_T C^2 \frac{1 + \cos(2\omega_0 t + \theta)}{2} dt$$

$$P_x = \frac{C^2}{2T}\int_0^T dt + \underbrace{\frac{C^2}{2T}\int_0^T \cos(2\omega_0 t+\theta)\,dt}_{0}$$

$$P_x = \frac{C^2}{2T}\cdot T = \frac{C^2}{2}$$

rms value: $X_{rms} = \left(\lim_{T\to\infty}\frac{1}{T}\int_T |x(t)|^2\,dt\right)^{1/2} = \sqrt{P}$

$X_{rms} = \frac{C}{\sqrt{2}}$ (rms = root of mean square value)

b) $P_x = \lim_{T\to\infty}\frac{1}{T}\int_T \left(C_1\cos(\omega_1 t+\theta_1) + C_2\cos(\omega_2 t+\theta_2)\right)^2 dt =$

$= \lim_{T\to\infty}\frac{1}{T}\int_T C_1^2\cos^2(\omega_1 t+\theta_1) + C_2^2\cos^2(\omega_2 t+\theta_2) + 2C_1C_2\cos(\omega_1 t+\theta_1)\cdot\cos(\omega_2 t+\theta_2)\,dt$

$= \lim_{T\to\infty}\frac{1}{T}\int_T C_1^2\cos^2(\omega_1 t+\theta_1)\,dt + \lim_{T\to\infty}\frac{1}{T}\int_T C_2^2\cos^2(\omega_2 t+\theta_2)\,dt +$

$\lim_{T\to\infty}\frac{1}{T}\int_T C_1\cos(\omega_1 t+\theta_1)\cdot C_2\cos(\omega_2 t+\theta_2)\,dt$

$= \frac{C_1^2}{2} + \frac{C_2^2}{2} + \lim_{T\to\infty}\frac{1}{T}\int_T \frac{C_1C_2}{2}\cos[(\omega_1-\omega_2)t+\theta_1-\theta_2]\,dt \nearrow 0 +$

$\lim_{T\to\infty}\frac{1}{T}\int_T \frac{C_1C_2}{2}\cos[(\omega_1+\omega_2)t+\theta_1+\theta_2]\,dt \searrow 0$

$= \frac{C_1^2}{2} + \frac{C_2^2}{2}$

$X_{rms} = \left[\frac{C_1^2}{2} + \frac{C_2^2}{2}\right]^{1/2}$

note: The result is easily extended to multiple sinusoidals.

Consider:

$$x(t) = \sum_{n=1}^{\infty} C_n \cos(\omega_n t + \theta_n) \quad , \quad \omega_n \neq 0$$

$$P_x = \frac{1}{2}\sum_{n=1}^{\infty} C_n^2 \qquad x_{rms} = \left[\frac{1}{2}\sum_{n=1}^{\infty} C_n^2\right]^{1/2}$$

c)

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_T |D e^{j\omega_0 t}|^2 dt$$

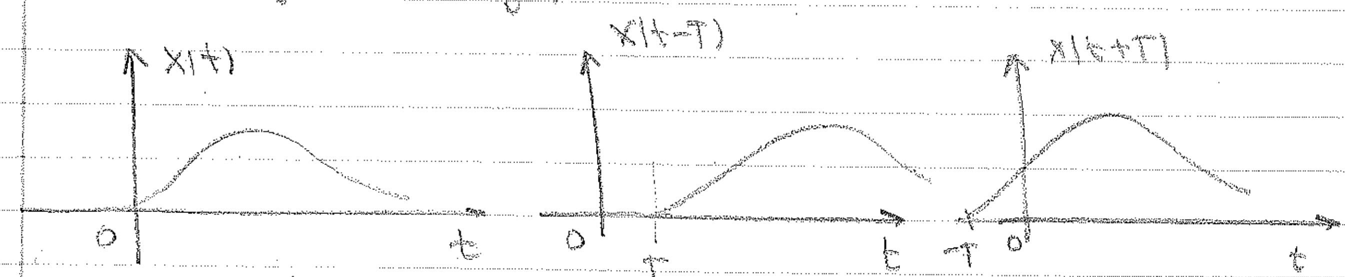
$$= \lim_{T\to\infty} \frac{1}{T}\int_T D^2\, dt = D^2$$

$$x_{rms} = \sqrt{P_x} = D$$

Some useful signal operations

1) Time shifting

$x(t)$ - signal
$x(t-T)$ - signal delayed by T
$x(t+T)$ - signal advanced by T



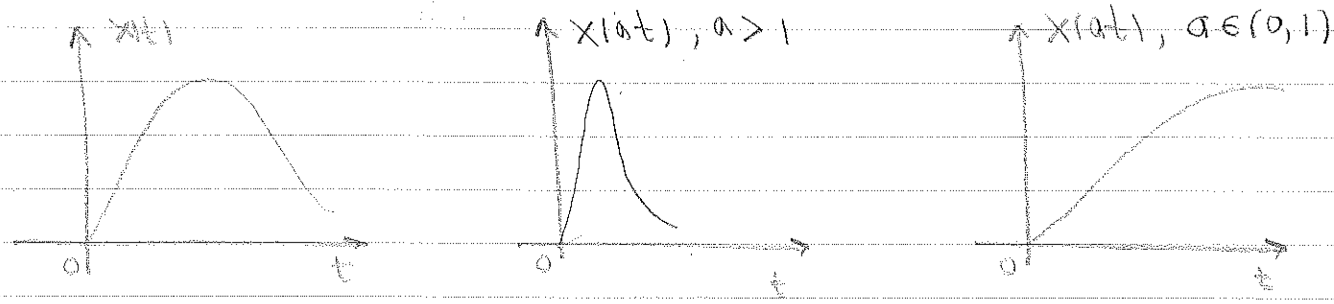
original signal   delayed by T advanced by T

2) Time scaling

$x(t)$ - signal

$x(at)$ - scaled signal $a > 1$ - "speeds" up signal

$0 < a < 1$ - "slows down" the signal



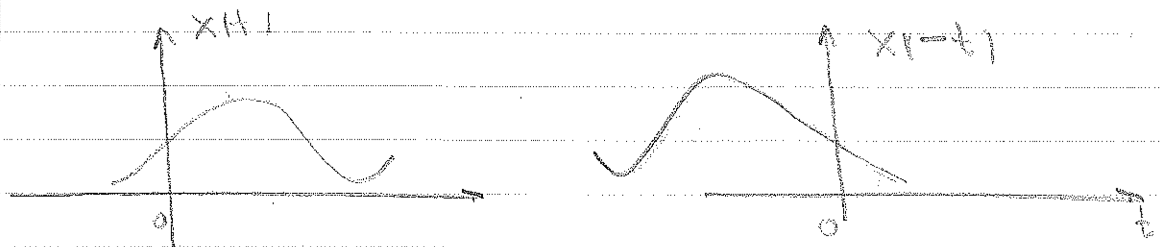
Original signal sped up version slowed down version

3) Time reversal

$x(t)$ - signal

$x(-t)$ - signal reversed in time

Time reversal of a signal - mirror image about y axis



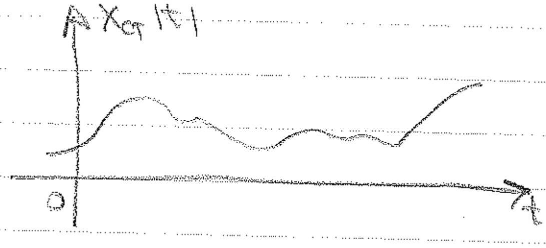
Classification of signals.

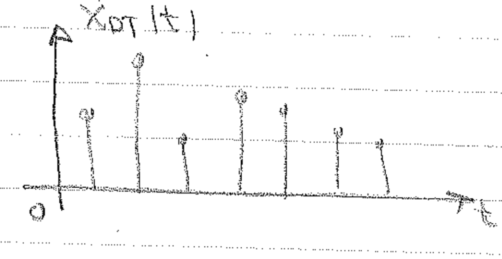
Signals may be classified as

1) Continuous time and discrete time
2) analog and digital
3) periodic and aperiodic
4) power and energy
5) deterministic and probabilistic

1) Continious time and discrete time

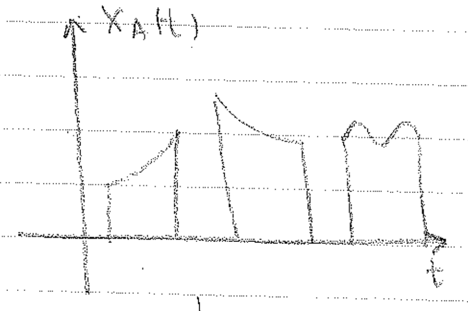
* continious time - defined for continious values of t
* discrete time - defined only in some instances of time



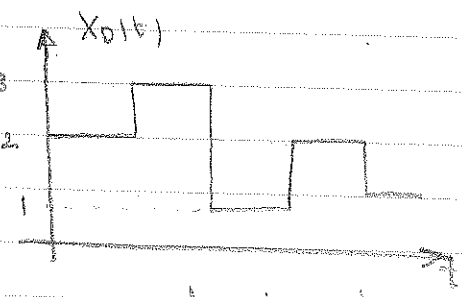


2) Analog and digital

* analog signals - take amplitude values from a continious range
* digital signals - discrete in amplitude domain. That is, the amplitude of the signal takes only one of N possible discrete values.



analog signal



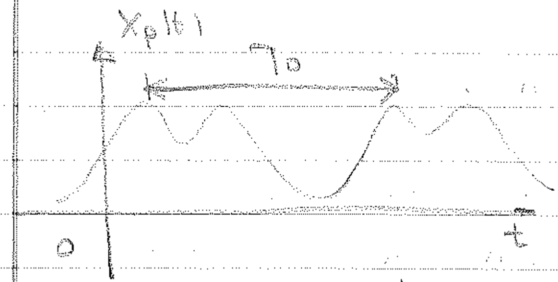
digital signal

3) Periodic and aperiodic signals

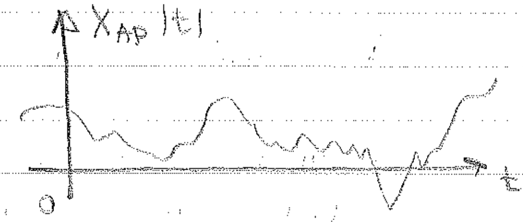
$x(t)$ - periodic signal if there exists a positive constant T_0 such that

$$x(t+T_0) = x(t), \forall t \quad (*)$$

The smallest value of T_0 that satisfies (*) is called signal period



periodic signal



aperiodic signal

4) Energy and Power signals

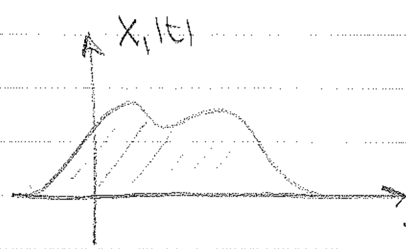
x(t) - signal

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt \quad \text{- energy}$$

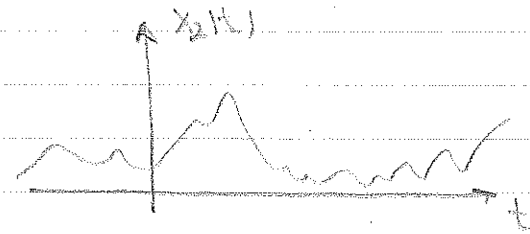
$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{+\infty} |x(t)|^2 dt \quad \text{- power}$$

note: all practical signals are either power or energy signals

Signal is an energy signal if: E_x - finite $P_x = 0$
signal is a power signal if: E_x - infinite P_x - finite



typical energy signal



typical power signal

5) Deterministic versus random signals

Deterministic signals - fully specified. Possible way of specifying the signals are: formula, table, graph, parameter set, ...

Random signals: - Specified only through their statistical properties. Random signals are specified by the statistic of the amplitude domain and statistics associated with the time domain.

1) amplitude domain - usually PDF of amplitude
2) time domain - usually PSD or Autocorrelation function

PDF - Probability Density Function
PSD - power spectrum Density

Selected problems.

1.1-1	1.2-1	1.3-1
1.1-3	1.2-3	1.3-3
1.1-4	1.2-5	1.3-5
1.1-6		1.3-6