(Lecture 14) Adaptive beamforming:

* One of the most popular applications of adaptive signal processing.
* Frequentlly implemented using LMS algorithm.
* Used for performance enhancement, interference cancellation, tracking, ...
* Many different methods.

Sidetone canceller:

0) Interferer

Rx

null of the antenna
pattern is placed in the
direction of interferer.

O) Original antenna pattern

Opportunistic scenario

1) RX is turned on
2) RX examines the RF environment and cancels all interferes
3) TX begins in interference reduced environment (usually TX is wideband, either FH or DS spread spectrum).

\[ n_{1}(k) \sim \mathcal{N}(0, \sigma_{n}) \]
\[ n_{2}(k) \sim \mathcal{N}(0, \sigma_{n}) \]
\[ n_{c}(k) \sim \mathcal{N}(0, \sigma_{n}) \]
\[ n_{c}(k) \& n_{c}(k) \Rightarrow \text{independent for all lags} \]
Signal in primary branch:

\[ d[k] = C e^{j \omega_0 k} + n_1[k] \]

\[ \omega_0 = \frac{f}{f_s}, \quad f_s = \text{sampling frequency} \]

\[ \omega_0 \in (0, 2\pi) \]

\[ n_1[k] - \text{sensed noise at the output of the primary branch.} \]

Signal in the reference branch:

\[ d[k] = C e^{j (\omega_0 + \theta_0) k} + n_2[k] \]

\[ \theta_0 = \text{phase shift of the sinusoidal that is due to} \]

\[ \theta = 2\pi \cdot \sin^2 \theta \]

\[ \frac{\pi}{2} \cos(\pi - \theta) = \frac{\theta}{2} \sin \theta \]

Drawn at the phase shift.

Let us consider the simplest case when there is only one interferer and only one weight in the FIR filter:

\[ y[k] = \omega^* u[k] = \omega^* u[k] = \omega^* [C e^{j (\omega_0 k + \theta)} + n_2[k]] \]

Therefore:

\[ e[k] = d[k] - y[k] = C e^{j \omega_0 k} + n_1[k] - \omega^* [C e^{j (\omega_0 k + \theta)} + n_2[k]] \]
\[ e(c) = (1 - \omega^* e^{i\theta}) c e^{j\omega t} + n_1(c)(1-\omega^* n_2(c)) \]

Where Riller approach,

\[ R = E[f \mu(c) u^*(c) f] = E[\int [c e^{j(\omega t + \theta)} + n_1(c)]] c e^{j(\omega t + \theta)} + n_2(c) n_1^*(c)f] \]

\[ = E\{ c^2 + c n_2(c) e^{j(\omega t + \theta)} + c n_1(c) e^{-j(\omega t + \theta)} + n_2(c) n_1^*(c) \} \]

\[ = c^2 + 0 + 0 + \delta n^2 = c^2 + \delta n^2 \]

\[ p = E[f \mu(c) \cdot d(c)] = E[\int c e^{j(\omega t + \theta)} + n_2(c) c e^{-j(\omega t + \theta)} + n_1(c) n_1^*(c) \} \]

\[ = E\{ c^2 e^{j\theta} + n_2(c) c e^{-j(\omega t + \theta)} + n_1(c) n_1^*(c) \} \]

\[ = c^2 e^{j\theta} \]

Therefore,

\[ \omega = R^{-1} p \Rightarrow \omega = \frac{1}{c^2 + \delta n^2} c^2 \cdot e^{j\theta} = \frac{c^2 / \delta n^2}{c^2 / \delta n^2 + 1} e^{j\theta} \]

\[ \omega = \frac{\text{SNR}}{1 + \text{SNR}} \cdot E[ e^{j\theta}] = \frac{\text{SNR}}{1 + \text{SNR}} (\cos \theta + j \sin \theta) \]

To obtain the optimal solution we then LMS algorithm as follows:

\[ R = \mu(c) u^*(c) \Rightarrow u(c) \cdot u^*(c) \]

\[ \beta = u(c) \cdot d^*(c) \Rightarrow u(c) \cdot d^*(c) \]

\[ \omega(0) = 0, \text{ set } \mu \]

\[ \omega(k+1) = \omega(k) + \mu (u(k) d^*(c) - u(k) u^*(c) \cdot \omega(k)) \]

\[ = \omega(k) + \mu u(k) e^*(c). \]
Let us examine antenna pattern of the system when the system is operating in optimum regime (the weights have converged). To understand operation of the system assume $SNR >> 1$.

\[ d(c) \rightarrow e(c) = z(k) \quad z(k) \text{ - output signal} \]

Input signal

\[ S_d(k) = C e^{j\omega d k} + n_i(k) \approx C e^{j\omega d k} \]

Desired signal

\[ S_r(k) = C e^{j(\omega_d k + \theta)} = C e^{j \omega_d k + 2\pi \frac{\theta}{\lambda} \sin \phi} \]

From figure:

\[ d(k) = S_p(k) = C e^{j\omega_d k} \]

\[ y(k) = S_r(k) \cdot \omega^x = C e^{j(\omega_d k + 2\pi \frac{\theta}{\lambda} \sin \phi)} e^{-j\theta} \]

\[ = C e^{j\omega_d k} \cdot C e^{j2\pi \frac{\theta}{\lambda} \left[ \sin \phi - \sin \phi_0 \right]} \]

Output signal

\[ e(k) = d(k) - y(k) = \]

\[ = C e^{j\omega_d k} \left[ 1 - e^{j2\pi \frac{\theta}{\lambda} \left[ \sin \phi - \sin \phi_0 \right]} \right] \]

\[ \sim \text{ antenna gain of the antenna system.} \]
\[ G(\theta) = \left| 1 - \epsilon j \sin \frac{\theta}{\lambda} (\sin \theta - \sin \theta_r) \right|^2 \]

To draw the pattern assume \( \theta_r = 0 \)

\[ G(\theta) = \left| 1 - \epsilon j \frac{\theta}{\lambda} \sin \theta \right|^2 = \left| 2j \epsilon \frac{\theta}{\lambda} \sin \theta \right|^2 = \frac{4 \epsilon^2 (\frac{\theta}{\lambda} \sin \theta)^2}{4j^2 \epsilon^2 (\frac{\theta}{\lambda} \sin \theta)^2} = 4 \sin^2 \left( \frac{\theta}{\lambda} \sin \theta \right) \]

\[ \delta(\theta) = 10 \log G(\theta) = 6 + 20 \log \left( \sin \left( \frac{\theta}{\lambda} \sin \theta \right) \right) \]

There is a null in the pattern. The antenna is aligned with the dipole.

When \( \text{SNR} \gg 1 \), the null is infinitely deep. This means that an interferer that is very strong, and sinusoidal, can be canceled.
If the SNR is not large

**Input signal**

\[ S_p(k) = C e^{j\omega_o k} + n_1(k) \]

**Reference signal**

\[ S_r(k) = C e^{j(\omega_0 k + 2\pi \frac{\theta}{\lambda} \sin \theta)} + n_2(k) \]

From Figure

\[ d(k) = S_p(k) = C e^{j\omega_0 k} + n_1(k) \]

\[ y(k) = S_r(k) \cdot \cos\theta = \left( C e^{j(\omega_0 k + 2\pi \frac{\theta}{\lambda} \sin \theta)} + n_2(k) \right) \frac{\text{SNR}}{1 + \text{SNR}} e^{-j\theta \frac{\pi}{2} \cdot \sin \theta} \]

\[ = C e^{j\omega_0 k} \frac{\text{SNR}}{1 + \text{SNR}} e^{j2\pi \frac{\theta}{\lambda} (\sin \theta - \sin \phi)} + \frac{n_2(k) \text{SNR}}{1 + \text{SNR}} e^{-j\theta \frac{\pi}{2} \cdot \sin \theta} \]

\[ z(k) = d(k) - y(k) = C e^{j\omega_0 k} \left[ 1 - \frac{\text{SNR}}{1 + \text{SNR}} e^{j2\pi \frac{\theta}{\lambda} (\sin \theta - \sin \phi)} \right] + \frac{n_2(k) \text{SNR}}{1 + \text{SNR}} e^{-j\theta \frac{\pi}{2} \cdot \sin \theta} \]

\[ = \frac{n_2(k) \text{SNR}}{1 + \text{SNR}} e^{-j\theta \frac{\pi}{2} \cdot \sin \theta} \]

\[ \text{antenna gain} \]

\[ \text{near focus} \]

\[ \text{SNR large} \]

\[ \text{SNR small} \]

\[ \text{Attenuation of the null} \]

\[ 20 \log \left( \frac{1}{1 + \text{SNR}} \right) = -20 \log (1 + \text{SNR}) \]