

Mathematical Model of the 5-DOF Sled Dynamics of an Electrodynamic Maglev System With a Passive Sled

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Abstract—A model that describes the five-degrees-of-freedom (5-DOF) dynamics of a passively levitated electrodynamic maglev system is presented. The model is based on the flux-current-force interactions and the geometric relationships between the levitation coils and the permanent magnets on the sled. The model is presented in a parametric state-space formulation, suitable to extract model parameters from input-output measurements in a minimum mean-square error sense. The proposed structure is very well suited for parameter estimation and to later develop robust feedback control of the sled dynamics.

Index Terms—Electrodynamic modeling, electrodynamic suspension, magnetic levitation, null-flux levitation.

I. INTRODUCTION

ELECTRODYNAMIC magnetic suspension systems (EDS maglev) are based on the repulsive forces acting on a magnet that moves in the magnetic field of a fixed coil. Permanent magnets are used instead of coils in the moving frame; therefore, no power is required in the levitated vehicle. In a passive EDS system, relative motion is required to achieve levitation, as well as an auxiliary system for take-off and landing. EDS maglev with a passive sled is the best choice in systems that require high reliability under extreme conditions. In theory, EDS maglev is inherently stable (the vertical forces tend to cancel deviations from the zero-flux line) and, therefore, inherently more robust than electromagnetic (EMS) maglev. Currently existing EDS systems are open-loop systems that rely on passive or hybrid damping schemes to improve flight stability. In practice, the lack of direct feedback control of the vehicle's trajectory has been a cause of poor performance and even instability. As speed increases, an open-loop EDS maglev system will eventually become unstable. Development of a robust, highly reliable EDS maglev suspension requires a thorough understanding of the relationships between coil currents,

levitation forces, and the multi-degrees-of-freedom (DOF) sled trajectory. This paper presents a parametric state-space model of the levitation dynamics of a null-flux system with a passive sled, well suited to develop robust feedback control at a later stage. The proposed model is based on the current-force-flux relationships between the magnets on the sled and the levitation coils. The following assumptions were made.

- 1) The component of the permanent magnets' magnetic field perpendicular to the plane of the coil (B_y , see Fig. 1) is assumed uniform, and the average field strength on each magnet is an unknown parameter to be estimated.
- 2) The mutual inductance between adjacent null-flux coils can be neglected. This assumption is supported by finite-element simulations of the magnetic field in adjacent coils as described in [1].
- 3) The flux through a coil depends only on the x and z (δ) position of a sled magnet with respect to that coil (see Fig. 1). All angles are assumed to be small.
- 4) The sled magnets stay always between the upper and lower edges of each figure-8 coil, which is guaranteed by the geometry of the track and the landing gear.
- 5) The magnetic field components in the plane of the coil are assumed to be functions of the x and z position of the sled magnet with respect to that coil (B_x and B_z , see Fig. 1).

The accuracy of the algebraic model can be improved by making the field density components B_y , $B_x(x, \delta)$, and $B_z(x, \delta)$ functions of the gap between the face of the magnet and the plane of the coil, y . However, the assumptions of the algebraic model are more accurate when the gap between the coil and the magnet is small. Also, computer simulations of the algebraic model predicted that $B_x(x, \delta)$ and $B_z(x, \delta)$ can be simplified to $B_x(\delta)$ and $B_z(x)$.

II. TECHNICAL BACKGROUND

Several models have been developed to describe the interaction between the levitation coils and the sled magnets in null-flux systems. He *et al.* [2], [5] developed models using dynamic circuit theory, where the track dynamics is represented by equivalent circuit equations in matrix form. Matrix parameters can include time and space dependencies, so these models can be highly accurate provided a robust method to estimate the time-varying parameters is available. The computing time required by this approach is, however, prohibitive for real-time applications considering the large number of parameters to be estimated. A simpler approach (Kent, [3], [4]) does not attempt

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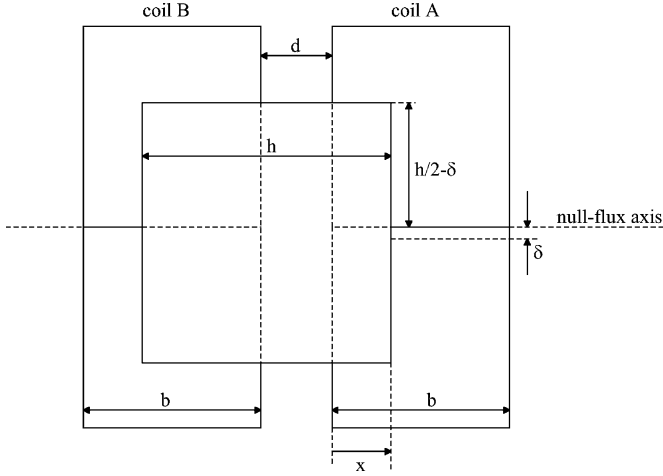


Fig. 1. Interaction of one magnet with two null-flux coils.

to analyze the entire system but only the interaction of a limited number of coils with a limited number of sled magnets. The analysis produces frequency-domain expressions which can be used to calculate time-averaged characteristics of the EDS maglev dynamics, very useful for design purposes but not suitable for real-time control. Neither of these approaches link the electrodynamic forces to the multi-DOF vehicle dynamics. This paper presents a mathematical model of the 5-DOF sled dynamics of a null-flux EDS system. The system uses a linear pulsed synchronous motor for propulsion and passive null-flux coils for levitation, making unnecessary an integrated analysis of suspension and propulsion (as described in [3], [5]) since both are physically decoupled by the way the track was built. The model allows off-line estimation of the required parameters and is simple enough to be used for real-time control.

III. INTERACTION OF A SLED MAGNET WITH TWO NULL-FLUX LEVITATION COILS

Fig. 1 shows the geometric interaction of one magnet with two null-flux coils, and the coordinates of the magnet relative to the coil.

The flux through each coil is approximated as

$$\begin{aligned}\Phi_A &= B_y x \left(\frac{h}{2} + \delta \right) - B_y x \left(\frac{h}{2} - \delta \right) \\ \Phi_A &= 2B_y x \delta\end{aligned}\quad (1)$$

$$\begin{aligned}\Phi_B &= B_y (b-x) \left(\frac{h}{2} + \delta \right) - B_y (b-x) \left(\frac{h}{2} - \delta \right) \\ \Phi_B &= 2B_y (b-x) \delta\end{aligned}\quad (2)$$

where x is the horizontal displacement of the magnet with respect to the left edge of coil A, B_y is the average magnetic field strength of the permanent magnet perpendicular to the coil (assumed constant) and δ is the vertical displacement of the center-line of the magnet with respect to the null-flux axis. In systems where $h = b + d$, the maximum value of x is $b + d$. After that, a new coil will start to interact with the magnet, becoming coil A. Then the former coil A becomes coil B and x must be reset

to zero. The flux equations change with the horizontal position x . For $b \leq x \leq h$, the flux equations are

$$\Phi_A = 2B_y b \delta \quad (3)$$

$$\Phi_B = 0. \quad (4)$$

In systems with a different geometry, it should be rather straightforward to develop a similar set of expressions. Similarly, equations can be derived for the flux through multiple levitation coils interacting with multiple magnets. These flux equations are used to calculate the induced voltages in the coils using the following expression, where N is the number of turns:

$$V = -N \frac{\partial \Phi_{\text{coil}}}{\partial t}. \quad (5)$$

For coils A and B, this becomes

$$V_A = -2NB_y(x\dot{\delta} - \delta\dot{x}) \quad 0 \leq x \leq b \quad (6)$$

$$V_A = -2NB_y b \dot{\delta} \quad b \leq x \leq h$$

$$V_B = -2NB_y(x\dot{\delta} + \delta\dot{x}) \quad 0 \leq x \leq b \quad (7)$$

$$V_B = 0 \quad b \leq x \leq h$$

The coil currents can be calculated from

$$\frac{\partial i}{\partial t} = \frac{1}{L}(V - Ri). \quad (8)$$

Coil currents can then be used to calculate the lift force acting on the magnet by using the Lorentz force equation

$$d\vec{F} = \vec{i} \times d\vec{B}. \quad (9)$$

The force acting on a particular magnet depends only on the coil current and the position of the magnet with respect to the coil. In this example, the following forces act on the magnet:

For $0 \leq x \leq b$

$$\begin{aligned}F_z &= i_a 2NB_y x + i_b 2NB_y (b-x) \\ F_y &= i_a 2N (B_x(x, \delta)\delta + B_z(x, \delta)x) + i_b 2NB_x(x, \delta)\delta \\ &\quad + i_b 2NB_z(x, \delta)(b-x) \\ F_x &= i_a 2NB_y \delta + i_b 2NB_y \delta.\end{aligned}\quad (10)$$

For $b \leq x \leq h$

$$\begin{aligned}F_z &= i_a 2NB_y b \\ F_y &= i_a 2N (B_x(x, \delta)\delta + B_z(x, \delta)xb) \\ F_x &= 0.\end{aligned}\quad (11)$$

The complete model of the sled and track interaction considers 20 figure-8 levitation coils in the track interacting with the 12 magnets on the sled (six on each side) at any given time, as shown in Fig. 2.

IV. 5-DOF MODEL OF THE SLED DYNAMICS

The 5-DOF model of the sled dynamics describes the motion of the sled as levitation height (z), sideways displacement (y), yaw (θ_1), pitch (θ_2), and roll (θ_3). The forward motion (x) is not considered since the propulsion dynamics are not part of the

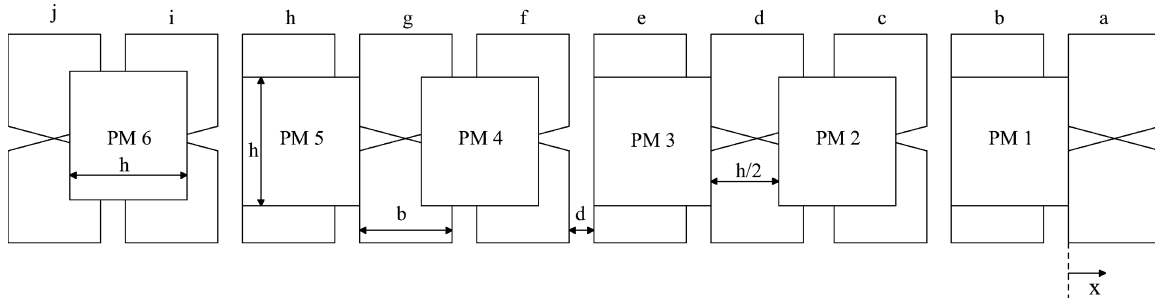


Fig. 2. Six magnets (one side of the sled) interacting with ten coils.

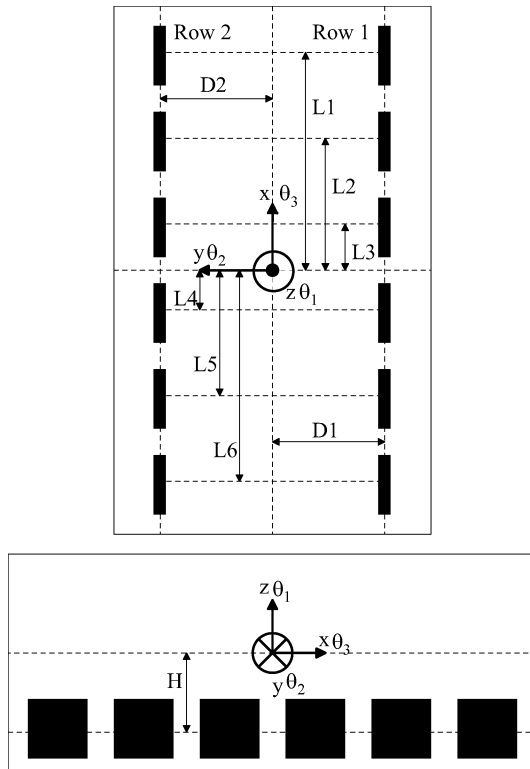


Fig. 3. Sled geometry with respect to the center of inertia.

model. The forces acting on each magnet are used to calculate all force components and torques. A disturbance torque is applied on yaw, pitch, and roll to model the unbalanced weight distribution on the sled.

Fig. 2 shows the layout of one side of the sled: each side has six permanent magnets that interact with ten levitation coils on the same side of track at any given time. The basic geometry of the magnets with respect to the sled's center of inertia is shown in Fig. 3.

To calculate the corresponding flux, voltage and force equations, four different sections must be considered:

- 1) $0 \leq x \leq (1/2)h - d$;
- 2) $(1/2)h - d \leq x \leq (1/2)h$;
- 3) $(1/2)h \leq x \leq b$;
- 4) $b \leq x \leq h$.

Each time x reaches the next coil ($x = h$), x is reset to 0. The coil indexes are then updated, so the new coil becomes coil A, A becomes B, B becomes C, and so on. All the electrodynamic forces acting on the sled are known, and the sled geometry is used to derive the motion equations for each DOF. Fig. 3

shows the basic sled's geometry relative to a coordinate system attached to the sled. The distances $L4$, $L5$, $L6$, and $D2$ and H have negative sign. The sled geometry relates the position and velocity of the magnets to the rigid body dynamics of the sled.

Kinematic relations for each sled magnet:

$$\begin{aligned} \delta_{ij} &= L_i \sin(\theta_2) + D_j \sin(\theta_3) \\ \dot{\delta}_{ij} &= L_i \dot{\theta}_2 \cos(\theta_2) + D_j \dot{\theta}_3 \sin(\theta_3) \end{aligned} \quad (12)$$

where $i = 1 : 6$ corresponds to the magnets along one side and $j = 1 : 2$ denotes right and left side, respectively. The force and torque equations are as follows.

Total lift force on sled:

$$\begin{aligned} F_{\text{lift}}^1 &= \sum_{i=1}^6 F_{z,i}^1 \\ F_{\text{lift}}^2 &= \sum_{i=1}^6 F_{z,i}^2 \\ F_{\text{lift}} &= F_{\text{lift}}^1 + F_{\text{lift}}^2. \end{aligned} \quad (13)$$

Total guidance force on sled:

$$\begin{aligned} F_{\text{guidance}}^1 &= \sum_{i=1}^6 -F_{y,i}^1 \\ F_{\text{guidance}}^2 &= \sum_{i=1}^6 F_{y,i}^2 \\ F_{\text{guidance}} &= F_{\text{guidance}}^1 + F_{\text{guidance}}^2. \end{aligned} \quad (14)$$

Total drag force on sled:

$$\begin{aligned} F_{\text{drag}}^1 &= \sum_{i=1}^6 F_{x,i}^1 \\ F_{\text{drag}}^2 &= \sum_{i=1}^6 F_{x,i}^2 \\ F_{\text{drag}} &= F_{\text{drag}}^1 + F_{\text{drag}}^2. \end{aligned} \quad (15)$$

Torques on sled:

$$\begin{aligned} T_{\theta_1} &= \sum_{i=1}^6 (-F_{y,i}^1 + F_{y,i}^2) L_i \cos(\theta_1) \\ &\quad + (F_{\text{drag}}^1 D_1 + F_{\text{drag}}^2 D_2) \cos(\theta_1) \\ T_{\theta_2} &= \sum_{i=1}^6 (F_{z,i}^1 + F_{z,i}^2) L_i \cos(\theta_2) \\ &\quad + F_{\text{drag}} H \cos(\theta_2). \end{aligned} \quad (16)$$

$$(17)$$

From these equations, the 5-DOF sled's motion equations can be derived. The x dynamics are not considered since

$$T_{\theta_3} = (F_{\text{lift}}^1 D1 + F_{\text{lift}}^2 D2) \cos(\theta_3) - F_{\text{guidance}} H \cos(\theta_3) \quad (18)$$

the propulsion dynamics are not estimated (i.e., x is directly measured)

$$\ddot{z} = \frac{F_{\text{lift}}}{M} - g \quad (19)$$

$$\ddot{y} = \frac{F_{\text{guidance}}}{M} \quad (20)$$

$$\ddot{\theta}_1 = \frac{T_{\theta_1}}{J_{\theta_1}} \quad (21)$$

$$\ddot{\theta}_2 = \frac{T_{\theta_2}}{J_{\theta_2}} \quad (22)$$

$$\ddot{\theta}_3 = \frac{T_{\theta_3}}{J_{\theta_3}}. \quad (23)$$

To take into account weight unbalance, constant disturbance torques were added to the angular dynamics. The moments of inertia were estimated using the sled's geometry and material properties. The model also includes some of the track's nonlinearities such as the rail's top, bottom, and sideways boundaries, which limit the trajectory.

A. Model Implementation

The model equations have been implemented in Matlab/Simulink as a user-defined C-code s-function. Its outputs are the 5-DOF trajectories and the currents induced in the levitation coils. Because of the complexity of the model, only the current of one coil at a time can be displayed. The model input is the x velocity during launch, and the corresponding velocity function can be chosen arbitrarily. Some of the model parameters have been estimated as accurately as possible, but the final goal is to extract model parameters from real-time measurements of the launch. See [9] for more information about parameters estimation in the 3-DOF case. Discontinuities in the plots are the result of the sled reaching the boundaries of the trajectories (the sled's dynamics is open loop at the moment).

V. SIMULATION RESULTS

An input speed profile that approximates the propulsion motion acting on the sled is shown on Fig. 4. The simulated trajectories for that specific input profile are shown in Figs. 5–9. All trajectories have poor damping and become unstable at sufficiently high horizontal (x) speeds. This behavior depends on the magnetic field strength as well. The mean pitch value tends to be negative during flight (the front of the sled is lower than the back). The yaw and sideways (y) dynamics seem unstable but this is partly a numeric artifact, since in theory the torque and force for yaw and sideways motion respectively should cancel, but round-off errors introduce residual torque and force. This has been verified by decreasing step size in the numerical solver.

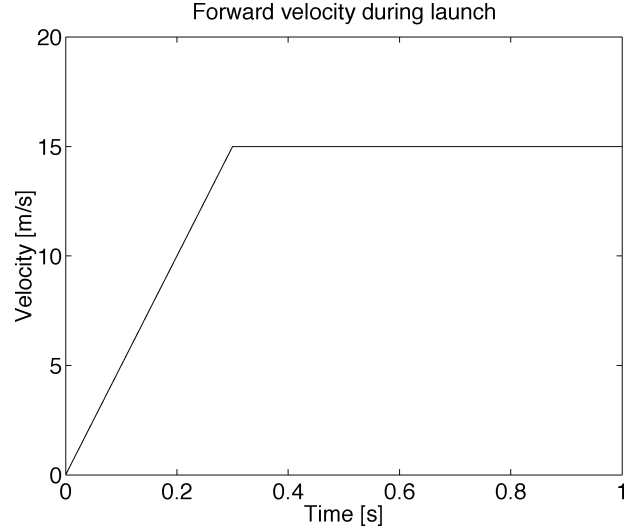


Fig. 4. Launch speed profile.

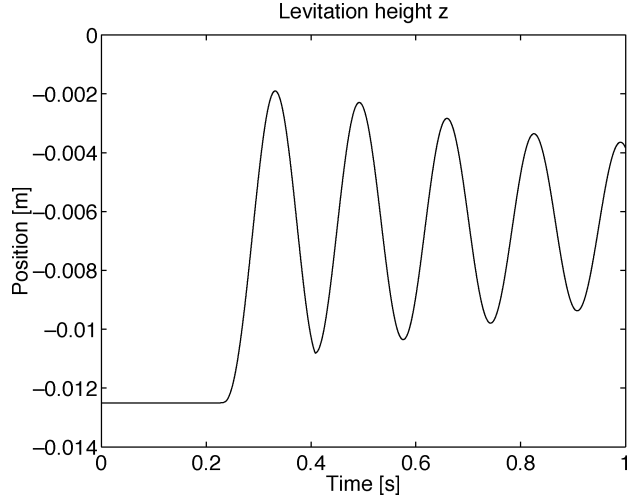


Fig. 5. Simulated z trajectory.

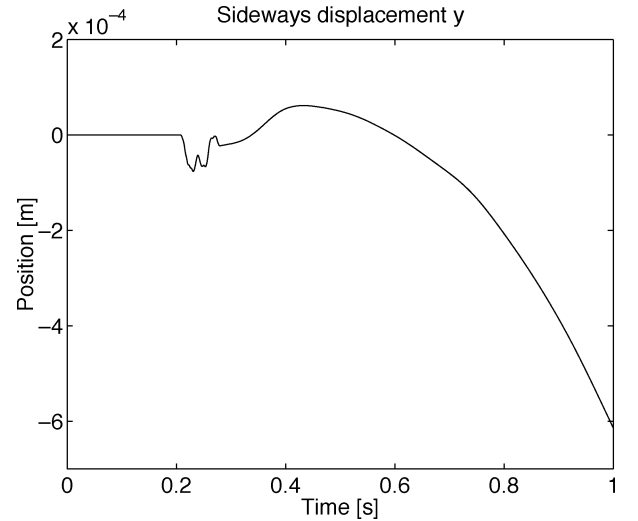


Fig. 6. Simulated y trajectory.

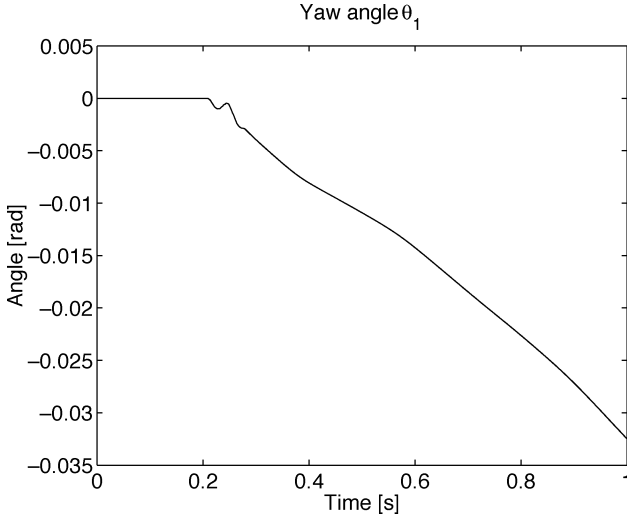
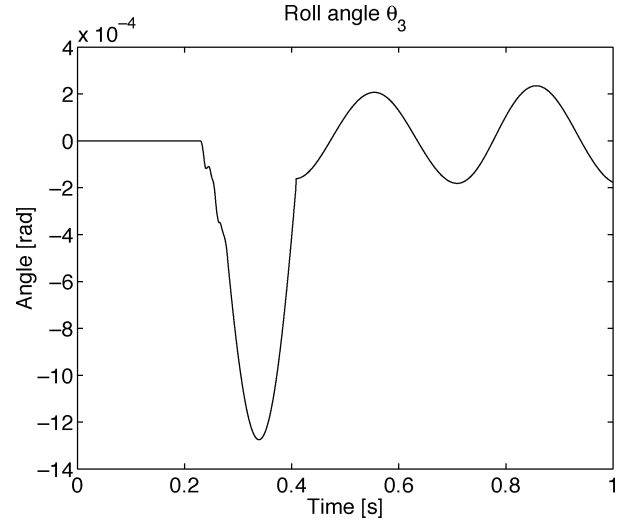
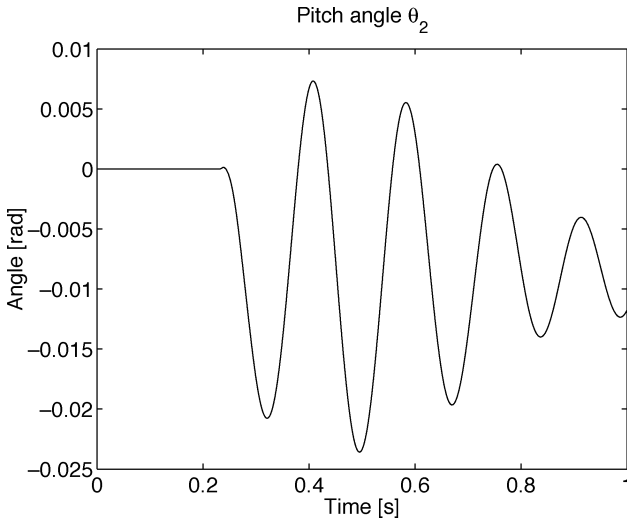
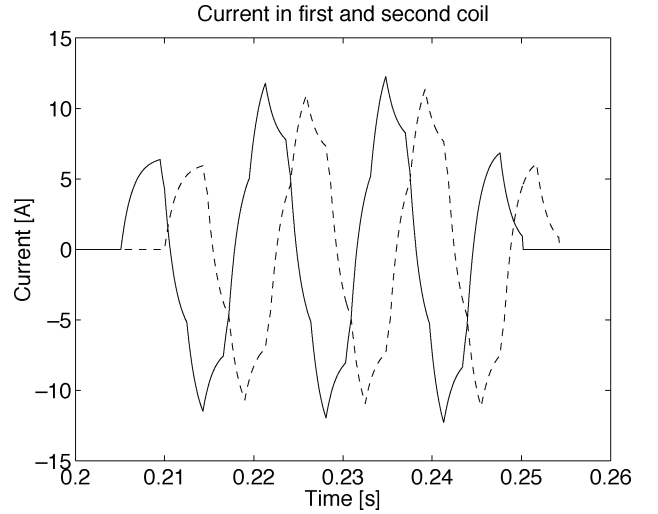
Fig. 7. Simulated θ_1 trajectory.Fig. 9. Simulated θ_3 trajectory.Fig. 8. Simulated θ_2 trajectory.

Fig. 10. Simulated current in the first two coils.

In Fig. 10, the simulated currents are shown for the first and second coils on the right side of the track.

VI. STATE-SPACE MODEL AND INPUT MAPPING

For future use in parameter estimation and real-time control, a state-space model is developed. It is assumed that all null-flux coil currents are inputs to the 5-DOF levitation dynamics. Since the Lorentz equation is used to calculate forces, all force components depend linearly on coil currents. This results in the following system:

$$\begin{aligned}\vec{x} &= [z \ y \ \theta_1 \ \theta_2 \ \theta_3 \ \dot{z} \ \dot{y} \ \dot{\theta}_1 \ \dot{\theta}_2 \ \dot{\theta}_3]^T \\ \vec{u} &= [i_a^1 \dots i_j^1 \ i_a^2 \dots i_j^2]^T \\ \dot{\vec{x}} &= A\vec{x} + B(x, \delta)\vec{u}\end{aligned}\quad (24)$$

where A is a 10×10 matrix consisting of purely inertial terms and B is a 10×20 input matrix. The first five rows of B are zeros and the elements in the last five rows are expressions of x

and δ and constant parameters. This system is overactuated and therefore difficult to control. Nevertheless, the proposed formulation is very useful for estimating the time-invariant system's parameters using a predictive error method and input-output measurements of trajectories and induced currents as is discussed in [9].

To formulate the control problem as an affine problem, an input mapping is proposed. Since the interaction of each coil with each DOF changes depending on the position of the sled, a constant input mapping is not possible. To introduce a time-dependent input mapping the B matrix of the state-space formulation is used. First, define

$$\begin{aligned}\vec{v} &= [v_z \ v_y \ v_{\theta_1} \ v_{\theta_2} \ v_{\theta_3}]^T \\ \vec{u} &= T(x, \delta)\vec{v} \\ \dot{\vec{x}} &= A\vec{x} + B(x, \delta)T(x, \delta)\vec{v} \\ B(x, \delta) &= \begin{bmatrix} O \\ P(x, \delta) \end{bmatrix}.\end{aligned}$$

The input mapping matrix T must be chosen to obtain the following structure:

$$\dot{\vec{x}} = A\vec{x} + \begin{bmatrix} O \\ H(x, \delta) \end{bmatrix} \vec{v} \quad (25)$$

where H is a square positive definite symmetric matrix. To attain this structure, the input mapping matrix T must be

$$T(x, \delta) = P(x, \delta)^T. \quad (26)$$

This choice of T ensures that the control action for each DOF will be distributed over all the coils according to their physical interaction. The structure of matrix H gives insight about the way the inputs are coupled

$$H(x, \delta) = \begin{bmatrix} c_{11} & 0 & 0 & c_{14} & 0 \\ 0 & c_{22} & c_{23} & 0 & c_{25} \\ 0 & c_{23} & c_{33} & 0 & c_{35} \\ c_{14} & 0 & 0 & c_{44} & 0 \\ 0 & c_{25} & c_{35} & 0 & c_{55} \end{bmatrix}. \quad (27)$$

The nonzero elements in H are all known functions of x and δ that can be calculated given that all constant parameters have been previously estimated and that the polynomial functions describing $B_x(x, \delta)$ and $B_z(x, \delta)$ in (10) and (11) are known. The structure of H also shows that pitch (θ_2) and levitation height (z) are coupled. The same is true for yaw (θ_1), roll (θ_3), and sideways displacement (y). The nonlinear functions for each element and the coupled dynamics suggest the use of a nonlinear control technique.

VII. FUTURE RESEARCH

A system identification experiment is proposed to extract optimal time-invariant model parameters in a minimum mean-square error sense. A schematic of the experimental setup is shown in Fig. 11. The coil driver boards can either generate computer-controlled test currents in the levitation coils, or be used to measure induced currents when in shorted mode. This makes possible to use the same setup to estimate parameters from trajectory measurements and measured currents as well as to implement real-time control. The relatively simple state-space formulation with input mapping is convenient for controller design, so a suitable controller strategy can be evaluated in simulation and implemented in real time.

VIII. CONCLUSION

A model structure that describes the 5-DOF dynamics of a levitated sled in a null-flux EDS system has been presented. The open-loop dynamics have poor damping and can be unstable at sufficiently high speeds, which agrees with previously published research [1], [2], [11]. The relative simplicity of the proposed structure allows the use of state-space notation, well suited for parameter estimation and later feedback control. Future research is aimed at the prediction of the sled's 5-DOF tra-

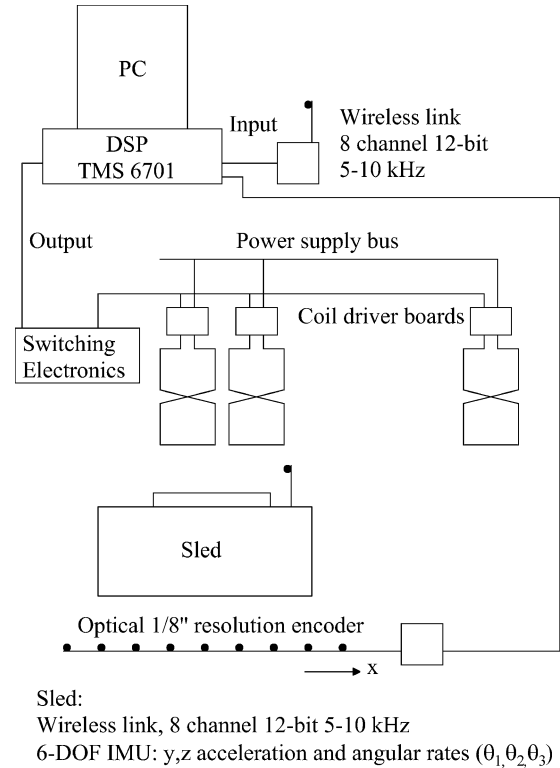


Fig. 11. Experimental setup.

jectories given a set of measured levitation currents and horizontal speed. The final goal is to achieve robust control of the 5-DOF sled's trajectories.

REFERENCES

- [1] J. Dill and D. Meeker, "Magfliter tradeoff study and subscale system demonstrations," Foster-Miller, Inc., Waltham, MA, NAS-98 069-1362, Dec. 2000.
- [2] J. L. He, D. M. Rote, and H. T. Coffey, "Applications of the dynamic circuit theory to maglev suspension systems," *IEEE Trans. Magn.*, vol. 29, no. 6, pp. 4153–4164, Nov. 1993.
- [3] R. D. Kent, "Designing with null flux coils," *IEEE Trans. Magn.*, vol. 33, no. 5, pp. 4327–4334, Sep. 1997.
- [4] —, "Analysis of an electro dynamic maglev system," *IEEE Trans. Magn.*, vol. 35, no. 5, pp. 4259–4267, Sep. 1999.
- [5] J. L. He, D. M. Rote, and H. T. Coffey, "Analysis of the combined maglev levitation propulsion and guidance system," *IEEE Trans. Magn.*, vol. 31, no. 2, pp. 981–987, Mar. 1995.
- [6] W. A. Jacobs, "Magnetic launch assist—NASA's vision for the future," *IEEE Trans. Magn.*, vol. 37, no. 1, pp. 55–57, Jan. 2001.
- [7] L. Ljung, *System Identification, Theory for the User*. Upper Saddle River, NJ: Prentice-Hall, 1987.
- [8] M. T. Thompson and R. D. Thornton, "Flux-canceling maglev suspension," *IEEE Trans. Magn.*, vol. 35, no. 3, pp. 1956–1975, May 1999.
- [9] J. De Boeij, H. M. Gutiérrez, R. Agarwal, and M. Steinbuch, "Modeling the 3-DOF dynamics of an electrodynamic maglev suspension system with a passive sled," in *Proc. 3rd Int. Conf. Transport Systems and Telematics*, Katowice, Poland, Nov. 2003.
- [10] J. De Boeij, H. M. Gutiérrez, and M. Steinbuch, "Modeling the electromechanical interactions in a null-flux EDS maglev system," in *Proc. 9th Int. Symp. Magnetic Bearings (ISMB9)*, S. Stephens, Ed., Lexington, KY, Aug. 2004.
- [11] Y. Cai, D. M. Rote, T. M. Mulcahy, Z. Wang, S. Chen, and S. Hsu, "Dynamic stability of repulsive-force maglev suspension systems," Argonne National Lab, Rep. ANL-96/18, Nov. 1996.