

# Modeling the Electromechanical Interactions in a Null-Flux Electrodynamical Maglev System

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**Abstract**—The fundamental electromechanical interactions in a passive null-flux electrodynamic maglev system (EDS) are mediated by the voltages induced in the levitation coils by the sled magnets, and by the forces exerted on the sled as a result of the induced currents. This paper presents a reliable and compact method to calculate these interactions by using analytical expressions of the magnetic field of a permanent magnet. The proposed approach provides a highly efficient and numerically stable approach to the computation of the flux induced in the levitation coils as well as the induced voltages and currents and lift, drag, and guidance forces acting on the sled's magnets. The analytical model is compared to a simplified algebraic model suitable for real-time control of EDS maglev suspension dynamics. Both models are compared with measurements and show good predictive quality.

**Index Terms**—Electrodynamical modeling, electrodynamic suspension, magnetic levitation, null-flux levitation.

## I. INTRODUCTION

DEVELOPMENT of a robust, reliable EDS maglev suspension requires a thorough understanding of the interaction between the magnets in the levitated vehicle and the null-flux coils. This paper presents a model of the voltages and currents induced in the null-flux coils of an open-loop EDS system as the launch vehicle runs along the track. From these, the forces acting on the vehicle are predicted.

Coupled electro-magneto-mechanical systems such as an EDS magnetic suspension have been typically modeled by finite-element methods (FEM), using numerical integration of Maxwell's equations to estimate the magnetic flux induced in the levitation coils. Currently existing packages that can solve the coupled electromagnetic, circuit, and motion equations demand a large computational effort and can be inaccurate due to limitations in mesh size and target speed.

In this paper, a closed-form representation of the magnetic field is used to calculate the induced flux, voltages, currents, and forces. This representation results in a computational method that is far simpler and more accurate than FEM methods, since

no numerical solution of the field differential equations is required.

Furthermore, a simplified algebraic model that is compact enough to be used in real-time control is presented and compared to the proposed analytical model. This simpler model results in algebraic equations for the flux through a null-flux coil and its time-derivative that can be solved even faster than the proposed analytical model. Both models are tested by comparing simulated model predictions with dynamic measurements of current and 3-axial force in a one-magnet, one-coil system.

## II. TECHNICAL BACKGROUND

Several models have been proposed to describe the interaction between the levitation coils and sled magnets in null-flux systems. He and Rote *et al.* [3], [4], [7] developed models using dynamic circuit theory, where the track dynamics are represented by equivalent circuit equations in matrix form. Matrix parameters can include time and space dependencies, so these models can be highly accurate provided a robust method to estimate the time-varying parameters is available. The computing time required by this approach is, however, prohibitive for real-time applications considering the large number of parameters to be estimated. A simpler approach proposed by Kent [5], [6] does not attempt to analyze the entire system but only the interaction of a limited number of coils with a limited number of sled magnets. The analysis produces frequency-domain equations which can be used to calculate time-averaged characteristics of the EDS maglev dynamics, very useful for design purposes but not suitable for real-time control.

A compact mathematical model developed by De Boeij and Gutiérrez [8] describes the three-degrees-of-freedom (3-DOF) sled dynamics of a null-flux EDS system by approximating the field of the sled magnets as time-invariant parameters to be estimated, and the field distribution as uniform over the levitation coils, hence estimating flux as a simple function of the geometry of the coils and the sled's position. The required parameters are estimated off-line from input-output data, and the model is simple enough to be useful for real-time control. This approach is nevertheless limited since it only calculates lift forces, and it cannot model the dependency of induced voltage and current with the distance between the plane of the magnets and the plane of the coils. In this paper, the model described in [8] is extended to calculate forces in all three directions and is compared to a proposed more accurate analytical closed-form model. Finally, both models are compared with measurements. Fast computation of the three force components acting on each one of the sled's magnets opens the path to real-time control of the multi-DOF suspension dynamics of an EDS maglev system.

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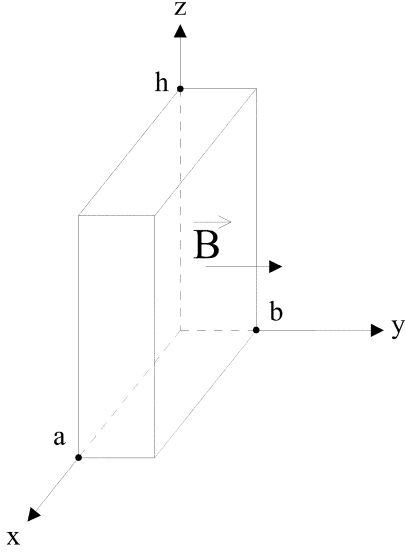


Fig. 1. Coordinate system for magnetic field equations.

### III. MODELING THE MAGNETIC FIELD AND LINKING FLUX

In this section, both the algebraic model and the analytical model are presented. The models describe the magnetic field of a sled magnet and the flux it produces in a null-flux levitation coil.

#### A. Analytical Model Based on Closed Form Equations of the Magnetic Field

Analytical equations exist that describe the magnetic field density caused by a permanent magnet in any point in space  $(x, y, z)$  as a function of the magnet geometry and material properties. In this case, a rectangular magnet is considered. The magnetic field can be computed [2] using (1)–(3).

The coil orientation is defined as parallel to the  $x$ - $z$  plane at a fixed distance  $y$  as shown in Figs. 1 and 2. To calculate flux, the coil's surface is divided in a number of area elements. In each element, the magnetic flux density perpendicular to the plane of the coil ( $B_y$ ) is calculated using (2) and (4). The flux linking each element is equal to the area of the element multiplied by the magnetic flux density  $B_y$ . In order to obtain accurate results, a sufficient number of elements must be used. The number of elements is increased until very small changes result. The flux induced by other magnets not directly in front of the coil is considered negligible, as well as the flux induced by the nearby coils (i.e., the mutual inductance of two adjacent coils is near zero). The last two assumptions are supported both by existing literature and FEM simulations [1].

#### B. Algebraic Model of the Electromagnetic Interactions

The algebraic model is based on the following assumptions.

- 1) The  $y$  component of the magnetic field of the permanent magnet is assumed uniform over its surface, with the magnetic field density ( $B_y$ ) a parameter to be estimated.
- 2) The flux through the levitation coil depends only on the  $x$  and  $z$  position of the magnet with respect to the coil. Angles are assumed to be small and of little influence.
- 3) The magnet stays always between the upper and lower edges of each null-flux coil.

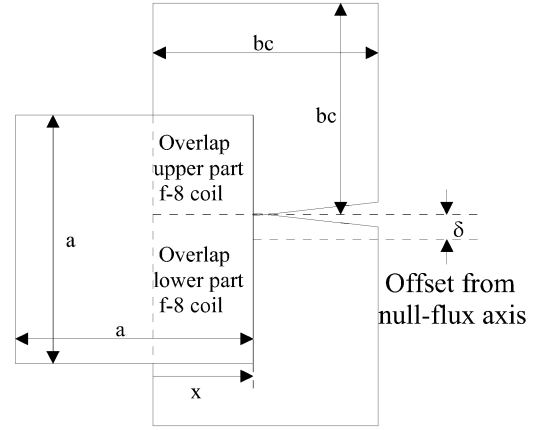


Fig. 2. Layout of a magnet passing a null-flux coil.

- 4) The magnetic field components in the plane of the coil and the surface of the magnet are assumed to be functions of the  $x$  and  $z$  position of a sled magnet with respect to that coil ( $B_x$  and  $B_z$ , see Fig. 1).

Using these assumptions, the flux linking the upper and lower part of the null-flux levitation coil can be defined as the overlap area of the coil and the magnet multiplied by the constant magnetic field density  $B_y$ . The net flux linking the coil is then calculated by subtracting the flux linking the lower part of the coil from the one linking the upper part of the coil.

If the position and velocity of one or multiple magnets are known in time, the total flux linking the null-flux coil and its time-derivative can be calculated using geometric relationships. The assumptions are necessary to keep the geometric relationships simple. The field components in the plane of the coil ( $B_x$  and  $B_z$ ) are fitted by polynomials of  $x$  and  $\delta$ . These functions are fitted in a least-square error sense from field values generated using the closed-form analytical equations.

#### C. Equations for Voltage and Current

The induced voltage in a short circuited null-flux coil can be calculated from (5), where  $\Phi$  is the net flux linking

$$H_x = \frac{-M}{4\pi} \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 (-1)^{i+j+k} \times \ln \left[ z_k + (x_i^2 + y_j^2 + z_k^2)^{1/2} \right] \quad (1)$$

$$H_y = \frac{-M}{4\pi} \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 (-1)^{i+j+k} \times \left\{ \tan^{-1} \left[ \frac{x_i y_j}{z_k (x_i^2 + y_j^2 + z_k^2)^{1/2}} \right] + \tan^{-1} \left[ \frac{y_j z_k}{x_i (x_i^2 + y_j^2 + z_k^2)^{1/2}} \right] \right\} \quad (2)$$

$$H_z = \frac{-M}{4\pi} \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 (-1)^{i+j+k} \times \ln \left[ x_i + (x_i^2 + y_j^2 + z_k^2)^{1/2} \right] \quad (3)$$

$$\begin{aligned} x_1 &= x & y_1 &= y & z_1 &= z \\ x_2 &= x - a & y_2 &= y - b & z_2 &= z - h \end{aligned} \quad M = |M|$$

$$\vec{B} = \mu_0 \vec{H} \quad (4)$$

the null-flux coil and  $N$  the number of turns

$$V = -N \frac{d\Phi}{dt}. \quad (5)$$

By solving the differential equation of the electrical circuit (6), the current in the coil is

$$\frac{di}{dt} = \frac{1}{L}(V - Ri) \quad (6)$$

where  $L$  is the inductance of the null-flux coil and  $R$  is its resistance.

#### D. Calculating Lorentz Forces

The Lorentz forces acting on the coil are calculated using the corresponding the magnetic field equations. The general form of the Lorentz force acting on an infinitely small wire is

$$d\vec{F} = i d\vec{l} \times \vec{B}. \quad (7)$$

Since the magnetic field component  $B_y$  in the algebraic model is approximated as constant over the coil's surface and the components  $B_x$  and  $B_z$  are polynomial functions of  $x$  and  $\delta$ , the forces in the *algebraic* model are computed using (8)–(10), where  $x$  is the length of the horizontal wire at the center of the coil that overlaps with the magnet. The net drag force is zero when the magnet overlaps the coil in the horizontal direction, since the drag force acting on the right wire cancels the force acting on the left wire

$$F_{\text{lift}} = 2NB_y ix \quad (8)$$

$$F_{\text{guidance}} = 2NB_x(x, \delta) i \delta + 2NB_z(x, \delta) ix \quad (9)$$

$$F_{\text{drag}} = 2B_y N i \delta. \quad (10)$$

Instead of these equations, the *analytical* closed-form model uses the general Lorentz force equation. Each coil loop is divided in differential segments of length  $dl$ , and the magnetic field density  $B$  is calculated at the center of each segment using (1)–(4). The force is then calculated using (7).

#### IV. EXPERIMENTAL SETUP

The experimental setup consists of a rotating arm with a null-flux coil attached to one end. The arm is driven by a servo controlled dc motor at constant speed. Induced current (or voltage) is measured and sampled at 32 kHz, and the data is sent to a computer in ground through a wireless transmitter attached to the arm. A permanent magnet in a frame fixed in ground is mounted facing the null-flux coil. The magnet frame is mounted to a triaxial force load cell. A laser displacement sensor is used to measure the offset of the magnet with respect to the null-flux axis of the figure-8 coil, which is defined as the height at which no net voltage is induced in the figure-8 coil. A schematic of the setup is shown in Fig. 3.

#### V. EXPERIMENTAL RESULTS

In this section, experimental results and model simulation results are presented. In each experiment, the distance to the

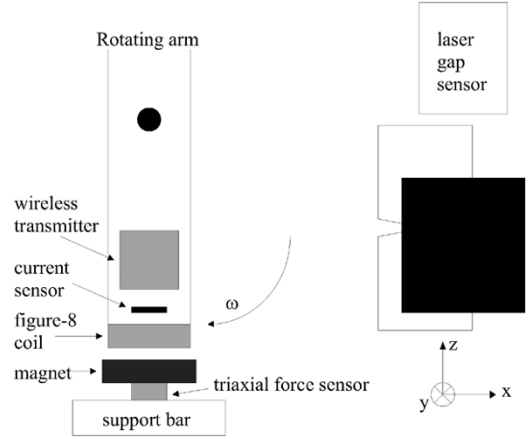


Fig. 3. Experimental setup for voltage, current, and force measurement.

TABLE I  
PARAMETER VALUES

Experiment	Parameter	Value
1	$\delta$	8.5 mm
	$g_y$	8.2 mm
	$v$	1.5 m/s
2	$\delta$	8.6 mm
	$g_y$	10.2 mm
	$v$	7.7 m/s
3	$\delta$	8.4 mm
	$g_y$	9.0 mm
	$v$	2.2 m/s

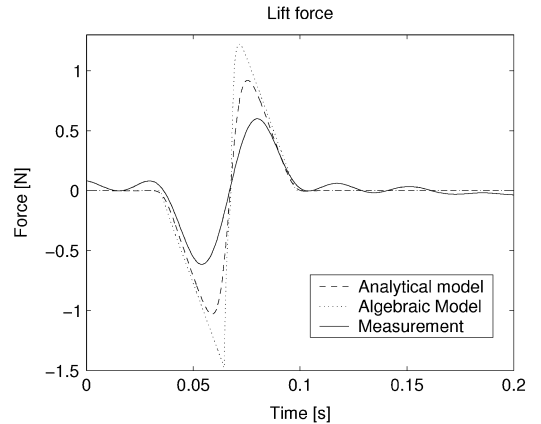


Fig. 4. Lift force experiment 1.

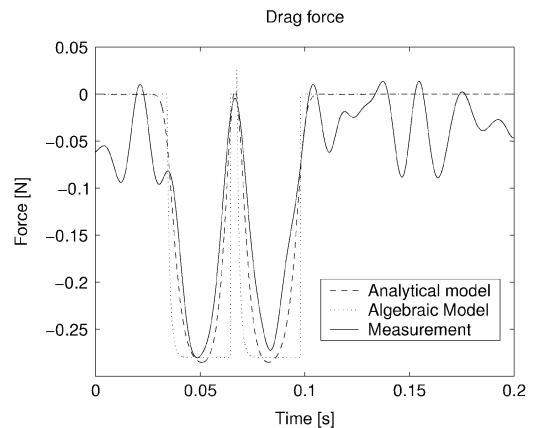


Fig. 5. Drag force experiment 1.

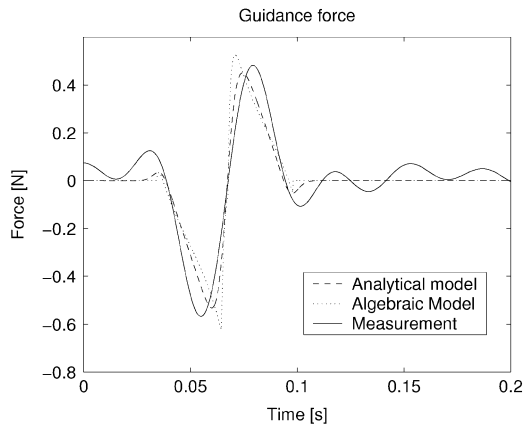


Fig. 6. Guidance force experiment 1.

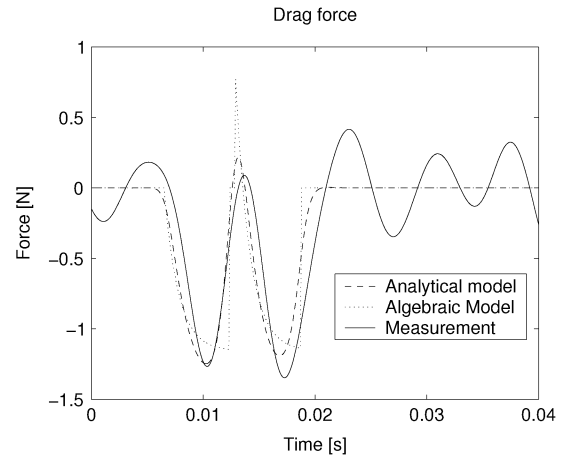


Fig. 9. Drag force experiment 2.

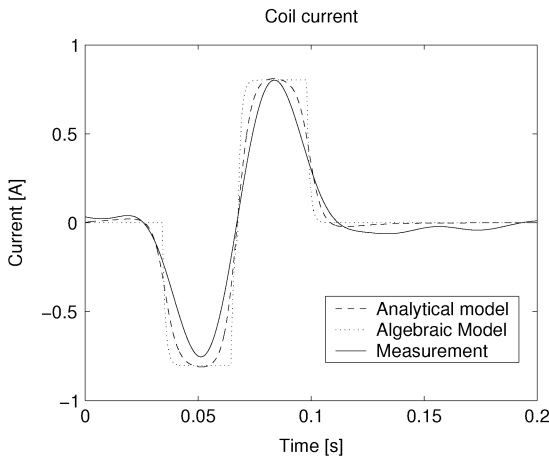


Fig. 7. Coil current experiment 1.

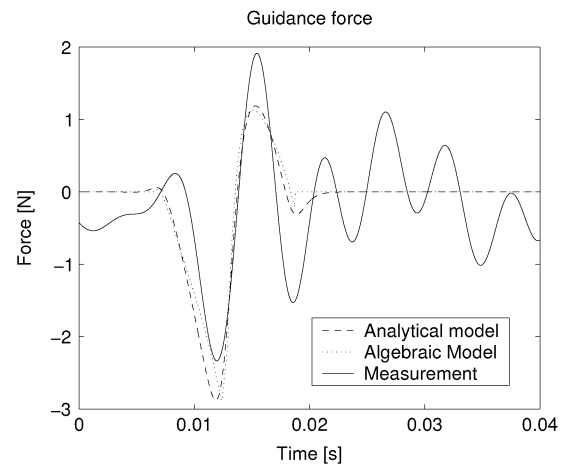


Fig. 10. Guidance force experiment 2.

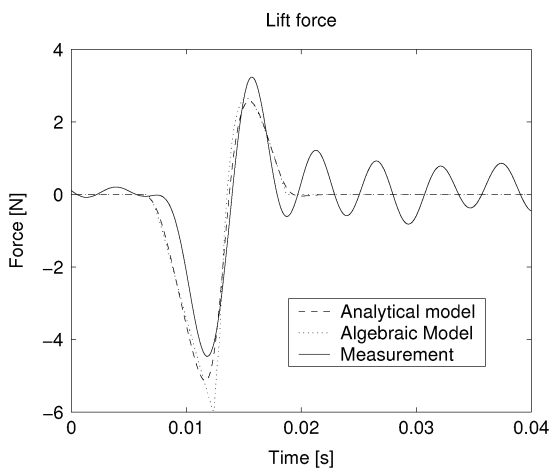


Fig. 8. Lift force experiment 2.

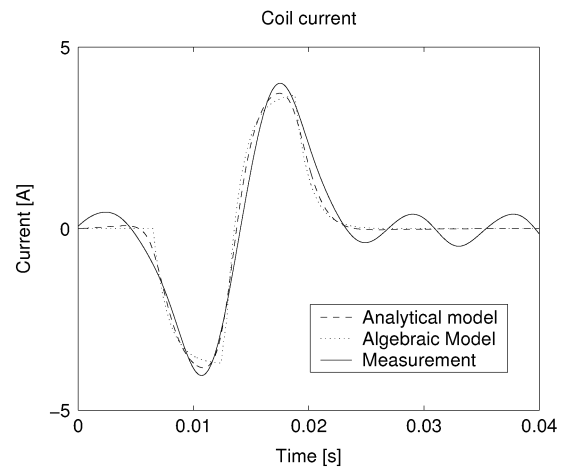


Fig. 11. Coil current experiment 2.

null-flux axis ( $\delta$ ), the speed of the rotating arm ( $v$ ) and the gap between the coil and the magnet ( $g_y$ ) are different. The parameters for each experiment are shown in Table I. Results for experiment 1 are presented in Figs. 4–7, experiment 2 in Figs. 8–11, and experiment 3 in Figs. 12–15.

## VI. DISCUSSION

The measured current is predicted accurately by both models in all three experiments. Predicted forces closely follow measured values, although structural vibrations that couple with force measurements (specially at high speeds) makes accurate force measurements difficult, i.e., large amplitude vibrations (Figs. 8–10) are due to structural vibrations of the test setup,

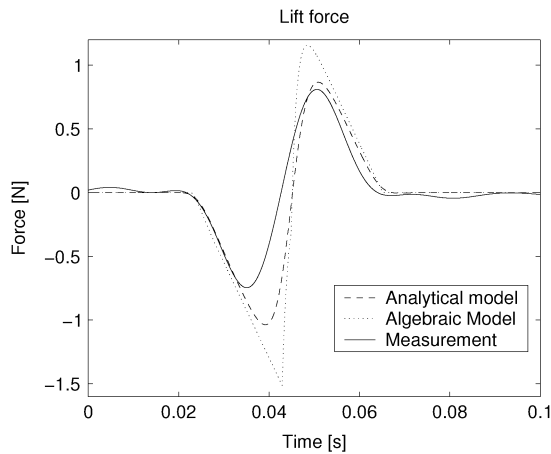


Fig. 12. Lift force experiment 3.

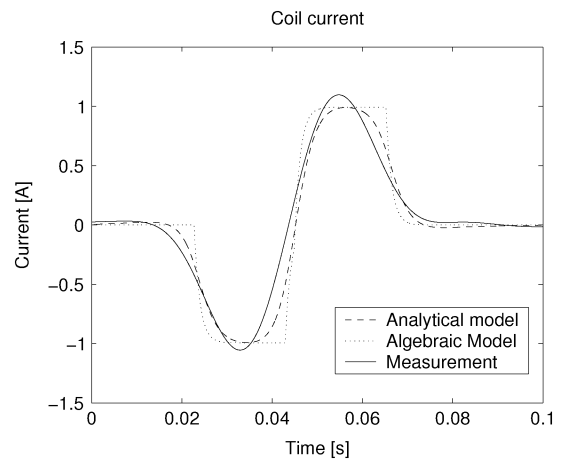


Fig. 15. Coil current experiment 3.

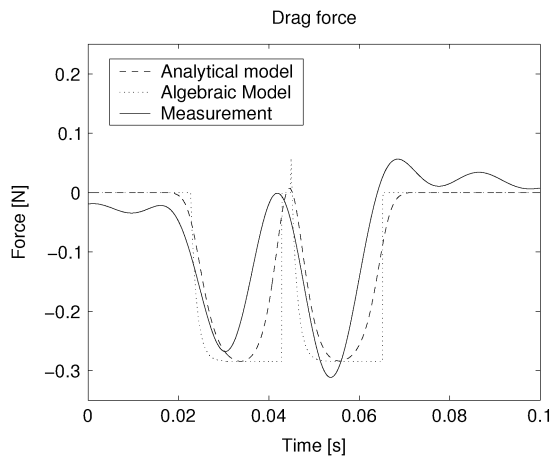


Fig. 13. Drag force experiment 3.

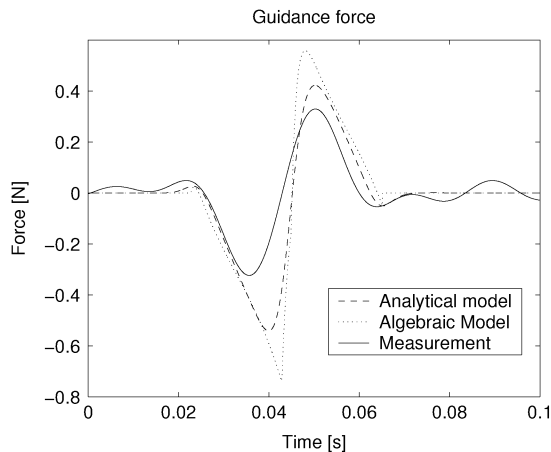


Fig. 14. Guidance force experiment 3.

and not a filtering artifact. According to the models, the two peaks in the drag force curve should have the same amplitude. The small difference in amplitude in the measurements is caused by the fact that the coil and the magnet are not perfectly parallel. Prediction errors from the algebraic model are related to the relatively large gap between the coil and the magnet, i.e.,

the prediction capability of the algebraic model improves when smaller gaps are considered, as discussed in [8]. The accuracy of the algebraic model can easily be improved by making the field density components  $B_y$ ,  $B_x(x, \delta)$ , and  $B_z(x, \delta)$  functions of the distance between the surface of the magnet and the plane of the coil,  $y$ . In any case, the assumptions of the algebraic model are more accurate when the distance between the coil and the magnet is small. Computer simulations of the algebraic model predicted also that  $B_x(x, \delta)$  and  $B_z(x, \delta)$  can be simplified to  $B_x(\delta)$  and  $B_z(x)$ .

## VII. CONCLUSION

Both the algebraic and the analytical closed-form model are capable of describing the electromechanical interactions in a null-flux EDS system. The models are much more computationally efficient than other existing methods such as FEM analysis and dynamic circuit theory. Comparisons between simulations and measurements show good agreement. The algebraic model is fast enough to be used in real-time control of the EDS maglev multi-DOF vehicle dynamics.

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