Magnetic Servo Levitation by Sliding-Mode Control of Nonaffine Systems With Algebraic Input Invertibility

Hector M. Gutierrez, *Member, IEEE,* and Paul I. Ro

***Abstract*—Magnetic Servo Levitation (MSL) is an important actuation principle with potential applications ranging from ultrahigh-precision positioning to high-speed rail systems. This paper describes a nonlinear controller design technique for MSL that has inherent robustness to both parametric uncertainties and unmodeled dynamics. Most of the currently available literature on sliding mode considers nonlinear systems that are linear (affine) in the input action. The proposed technique allows designing sliding-mode controllers for the family of nonaffine problems that have an input nonlinearity algebraically invertible with respect to the available control action. This differs from the standard approach of input feedback linearization, and is based on a modified sliding condition that can be used to synthesize a switching control law. An equivalent control term can also be included, substantially enhancing the performance of the controller. Experimental results show that the proposed technique can achieve excellent tracking at high speeds in a fast-tool servo system actuated by MSL.**

***Index Terms*—Equivalent control, fast-tool servo system, input feedback linearization, magnetic servo levitation, nonaffine nonlinear systems, sliding-mode control.**

I. Introduction

MAGNETIC Servo Levitation (MSL) is a novel actuation principle that has the potential to have a significant impact in both ultrahigh-precision and high-speed rail systems by providing frictionless interaction of moving parts, very high positioning accuracy at high speeds, high resolution and repeatability, and dust-free operation. MSL systems are virtually maintenance-free because they are not subject to wear, are mechanically simple and easy to assemble and manufacture since they require less demanding tolerances and can provide higher accelerations and velocities than conventional actuator technologies.

There are, however, several unresolved issues in the use of magnetic servo levitation, which have spurred substantial amount of research in the last few decades. While significant advances have been made in magnetic levitation, magnetic bearings, and advanced positioning technology, a unified approach to the design and control of MSL systems is still nonexistent.

The limitations in the use of MSL are related to the lack of adequate mathematical models and control algorithms that can take full advantage of its inherent potential. Magnetic servo control is a complex problem since the levitation force is a nontrivial function of the magnetic flux and hence depends in a complex fashion on device geometry, circuit parameters and material properties. The computational complexity involved in accurately calculating or estimating magnetic flux makes its computation impractical in real time, prompting most currently available control algorithms to use simplified models of the voltage-flux-force relationships, significantly limiting the overall performance of the system.

A number of control algorithms for magnetically levitated servo systems have been proposed [7], [8], [14]. Sliding-mode control is generally regarded as a reliable approach, for its robustness to process disturbances and parametric uncertainties. However, the standard sliding-mode formulation [1], [3]–[5] is restricted to systems that are affine (linear) in the input, which MSL systems are not. This limitation is generally addressed by linearizing the input by an algebraic transformation (feedback linearization), which in practice can severely limit performance due to the nonrobust nature of a model-dependent transformation [11].

This paper presents a novel control algorithm for MSL that is based on a modified sliding condition. The technique addresses nonaffine nonlinear systems with an algebraically invertible input nonlinearity, so that a nonrobust input transformation is not necessary. Experimental results show that excellent tracking performance can be achieved even at high speeds, with low power consumption and moderate chatter.

II. Sliding-Mode Control—An Overview

Sliding-mode design [1]–[6] has several features that makes it an attractive technique to solve tracking problems in electromechanical nonlinear systems, the most important being its robustness to process disturbances and unmodeled dynamics, and the computational simplicity of the algorithm, which allows real-time implementation at high closed-loop rates. One way of looking at sliding-mode control is to think of the design process as a two-step procedure. First, a region of the state space where the system behaves as desired is defined (sliding surface design). Then, a control action that takes the system into such surface and keeps it there is to be determined. Robustness is usually achieved by a switching control law. Design of the control action can be based on different strategies, a straightforward one being to define a condition that makes the sliding surface an attractive region for the state vector trajectories. A brief overview of the standard sliding-mode formulation is

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H. M. Gutierrez is with the Department of Mechanical and Aerospace Engineering, Florida Institute of Technology, Melbourne, FL 32901 USA (e-mail: hgutier@fit.edu).

P. I. Ro is with the Department of Mechanical and Aerospace Engineering, North Carolina State University, Raleigh, NC 27695-7918 USA.

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presented in this section, to provide a necessary context for the proposed (modified) sliding algorithm. Consider a nonlinear system of the form

$$x_i^{(n_i)} = f_i(\vec{x}) + \sum_{j=1}^{m} b_{ij}(\vec{x}) u_j, \qquad i = 1, \ldots, m, j = 1, \ldots, m \tag{1}$$

where $\vec{u} = [u_1, \ldots, u_m]^T$ is the vector of m control inputs, and the state vector $\vec{x}$ is composed of the x_i coordinates to be tracked and their first $(n_i - 1)$ derivatives. Such systems are called square systems since they have as many control inputs u_j as outputs to be controlled x_i [3]. The tracking problem consists of making the state vector $\vec{x}$ track a desired trajectory $\vec{r}$. Consider first the time-varying manifold σ given by the intersection of the surfaces $s_i(\vec{x}, t) = 0$, $i = 1, \ldots, m$, specified by the components of $S(\vec{x}, t) = [s_1(\vec{x}, t), \ldots, s_m(\vec{x}, t)]^T$, where

$$\begin{aligned} s_i(\vec{x}, t) &= \left(\frac{d}{dt} + \lambda_i\right)^{n_i - 1} (x_i - r_i) \\ &= x_i^{(n_i - 1)} + \cdots - r_i^{(n_i - 1)} \\ &= 0 \end{aligned} \tag{2}$$

which can be computed from $\vec{x}$ and $\vec{r}$, where λ_ι is some positive constant. Such manifold is usually called *sliding surface*, and any state trajectory lying on it tracks the reference $\vec{r}$ since (2) defines a differential equation on the error vector that stably converges to zero. An integral term can be incorporated into the sliding surface to further penalize tracking error. For instance, (2) can be rewritten for $n_i = 3$ as

$$\begin{aligned} s_i(\vec{x}, t) &= \left(\frac{d}{dt} + \lambda_i\right)^2 \left(\int_0^t (x_i - r_i) dt\right) \\ &= (\dot{x}_i - \dot{r}_i) + 2\lambda_i (x_i - r_i) + \lambda_i^2 \int_0^t (x_i - r_i) dt \\ &= 0. \end{aligned} \tag{3}$$

There are also several possible strategies to design the control action u_j that takes the system to the sliding surface. One such strategy is to find u_j in a way that each component of the sliding surface s_i is an attractive region of the state space by forcing the control action to satisfy a geometric condition such as

$$\frac{1}{2} \frac{d}{dt}(s_i^2) \leq -\eta_i |s_i| \Leftrightarrow s_i \frac{ds_i}{dt} \leq -\eta_i |s_i| \leq 0 \tag{4}$$

where η_i is a strictly positive constant. This condition forces the squared distance to the surface (as measured by s_i^2) to decrease along all state trajectories [3]. In other words, all state trajectories are constrained to point toward σ.

III. Sliding-Mode Control of Nonaffine Systems

Current literature on sliding mode deals almost exclusively with systems that are affine in the input, i.e., $\vec{x}^{(n)} = f(\vec{x}) + B(\vec{x})\vec{u}$, where $\vec{u}$ is the control input and f and B are nonlinear functions of the state vector. Under this formulation, systems with an input nonlinearity are usually approached by introducing a nonlinear coordinate transformation that renders the system affine [3], [5] or by small-signal linearization of the input [14]. The problem with the first approach is the inherent nonrobustness of the transformations involved (which are model-dependent): by making a coordinate transformation outside of the sliding control loop, the inverse transformation required to calculate the actual control action remains vulnerable to parametric uncertainty, often causing instability, poor performance, or both [15]. The second approach is additionally affected by the typical range and performance limitations of small-signal analyses.

Another approach has been to choose a set of state equations in which the system is affine in the input [7]. While it is often possible to "linearize" a nonlinear input relationship by including additional state equations, the problem once again is robustness: additional state equations usually mean additional uncertain parameters and modeling error. In high-precision applications, this severely affects overall system performance [15]. The approach proposed here, inspired by [12], addresses the limitations mentioned above by avoiding the use additional internal states and by using the robustness of a sliding-mode algorithm to overcome the modeling error introduced by the nonlinear input transformation.

Consider a decoupled nonlinear system that is nonaffine in the input

$$x_i^{(n_i)} = f_i(\vec{x}) + b_{ii} h_i(u_i, \vec{x}), \qquad i = 1, \ldots, m \tag{5}$$

where $\vec{x} = [x_1, \ldots, x_m]^T$ is the vector of m states, $B_{mxm} = diag(b_{11}, \ldots, b_{mm})$ is a diagonal matrix of uncertain parameters with known bounds, and each component of the input nonlinearity $\vec{h}_{mx1}(\vec{u}, \vec{x}) = [h_1(u_1, \vec{x}) \ldots h_m(u_m, \vec{x})]^T$ is a function of a single coordinate in the input space algebraically invertible with respect to a scalar upper bound $n(\vec{x})$, i.e., $\exists h_k^{-1}(\vec{x})/u_k = h_k^{-1}(\vec{x}, n(\vec{x}))$, $k = 1, \ldots, m$, where $0 < |\rho_j(\vec{x})| < n(\vec{x})$, $j = 1, \ldots, m$, where $\vec{\rho}_{mx1}(\vec{x})$ is a function of the desired sliding surface $S_{mx1}(\vec{x}, t)$ and the state equations $f(\vec{x})$. This formulation addresses an important number of practical applications that are nonaffine in the input: multiple degree-of-freedom (DOF) magnetic servo-levitated systems, autonomous underwater platforms, and automobile active suspensions.

A corresponding first-order sliding surface is defined by the vector of states to be tracked, $\vec{y}$, and the vector of desired trajectories, $\vec{r}$:

$$S_{mx1}(\vec{x}, t) = (\dot{\vec{y}} - \dot{\vec{r}}) + \Lambda(\vec{y} - \vec{r}); \vec{y} = C\vec{x}, \ \vec{y} \in R^m \tag{6}$$

where $\Lambda = diag(\lambda_1, \ldots, \lambda_m)$. The sliding condition (4) can be rewritten as

$$S^T \frac{dS}{dt} = S^T \left(\frac{\partial S}{\partial \vec{y}} \vec{f} + \frac{\partial S}{\partial \vec{y}} B \vec{h}(\vec{u}, \vec{x}) + \frac{\partial S}{\partial t}\right) < 0 \tag{7}$$

where $\vec{f} = [f_1(\vec{x}), \ldots, f_m(\vec{x})]^T$ is given by (5). We thus define

$$D_{mxm} = \frac{\partial S}{\partial \vec{y}} B \quad S^* = D^T S \Leftrightarrow S^T = (S^*)^T D^{-1} \tag{8}$$

$$\rho_{mx1}(\vec{x}) = D^{-1} \left(\frac{\partial S}{\partial \vec{y}} \vec{f} + \frac{\partial S}{\partial t}\right). \tag{9}$$

By replacing (8) and (9) in (7), a modified sliding condition equivalent to (4) can be obtained, assuming D in (8) is nonsingular

$$(S^*)^T\left(\vec{h}(\vec{u},\vec{x})+\rho(\vec{x})\right)<0. \tag{10}$$

Any control algorithm that satisfies this condition takes the system to sliding. The condition (10) is typically implemented as a switching control law. The proposed formulation covers an important family of nonlinear problems that are nonaffine in the input and whose dynamics are input-decoupled as in (5).

To estimate how uncertainties and external disturbances affect control chattering, it will be first assumed that they all satisfy the so-called matching condition [5] and can therefore be lumped into a single vector function ξ so that

$$x_i^{(n_i)}=f_i(\vec{x})+b_{ii}h_i(u_i,\vec{x})+b_{ii}\xi_i(u_i,\vec{x},\vec{q},\vec{d}), \qquad i=1,\ldots,m \tag{11}$$

where $\vec{q}$ is a vector function representing parameter uncertainties and $\vec{d}$ is a vector of external deterministic disturbances. Assuming also that $\|\xi\|$ is bounded by some positive continuous function $\tau(\vec{x},t)$, a Lyapunov-like sliding condition such as (4) can be used in (11) to find a switching control law that takes the system with uncertainties (11) into sliding, namely

$$u_{sw}=\begin{cases}-\dfrac{B^T\left(\frac{\partial S}{\partial \vec{x}}\right)^T S}{\left\|B^T\left(\frac{\partial S}{\partial \vec{x}}\right)^T S\right\|}\left[\tau(\vec{x},t)+\alpha(\vec{x},t)\right], & S(\vec{x},t)\neq 0\\ 0, & \text{otherwise}\end{cases} \tag{12}$$

where B is the same as in (5) and $\alpha(\vec{x},t)$ is to be determined based on satisfying the sliding condition. Equation (12) therefore illustrates how the boundary function τ of the uncertainty vector ξ affects control chatter and power consumption, since u_{sw} is part of the total control action.

The effect of external disturbances in tracking error can also be estimated from (12) considering the fact that infinitely fast switching is not possible and, therefore, any practical application of a switching control law implies a finite boundary layer. A boundary layer is a modification of the switching control law that alleviates chatter by defining smoother transitions of control command as opposed to infinitely fast switching, i.e., within a vicinity of thickness Φ of the sliding surface, the control output is scaled proportionally to the ratio (S/Φ). In a system where the states are the derivatives of the first state, i.e., $\vec{x}=\left[x\ \dot{x}\ \ldots\ x^{(n-1)}\right]^T$, the magnitude of the tracking error ε is bounded by a function of the boundary layer thickness Φ and the sliding surface parameter λ

$$\|\vec{x}-\vec{r}\|\leq\frac{\Phi}{\lambda^{n-1}}=\varepsilon. \tag{13}$$

If the system's input–output dynamics are given by (1) with a possibly time-varying or state-dependent control gain $B=\{b_{ij}\}$, the scalar bounds in the norm of B under some arbitrary metric define a ratio that can be called the *gain margin*, β [3]

$$0<b_{\min}\leq\|B\|\leq b_{\max} \qquad \beta=\left(\frac{b_{\max}}{b_{\min}}\right)^{1/2}. \tag{14}$$

In this kind of system [3], the tracking error ε can be estimated as

$$\varepsilon\approx\frac{\beta k(\vec{r})}{\lambda^n} \tag{15}$$

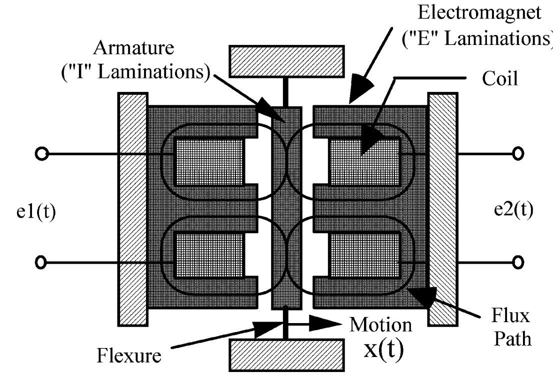

Fig. 1. Electromagnetic actuator based on attractive force.

where $k(\vec{x})$ is the gain of the switching term in the control law $u=u_{eq}+k(\vec{x})u_{sw}$. This illustrates the relationship between disturbances $\vec{d}$ and tracking performance, since $\vec{d}$ and u_{sw} are related by (12). Another aspect to consider is the fact that a control law such as (12) that derives from sliding conditions such as (4) or (10) requires full knowledge of the state vector $\vec{x}$, when actually it is only practical to measure some components of $\vec{x}$. The design of a nonlinear observer is in principle similar to that of a Luenberger observer [5]; a more complete discussion of nonlinear observers and the error dynamics of the state estimation is made by Misawa and Hedrick in [18].

IV. Magnetic Servo Levitation for High-Precision Positioning

MSL has been investigated [8]–[11] as an actuation principle to drive ultrahigh-precision positioning systems such as active tool holders (fast-tool servo systems) that could overcome the range limitations of piezoelectric-driven devices while operating over a wide frequency band. A 1-DOF MSL actuator (Fig. 1) is an electromagnetic device based on attractive forces, as opposed to the Lorentz (shear) forces used by linear motors. Very large forces are available as the gap becomes smaller, but the range of motion is small and the system is inherently unstable and highly nonlinear. To control such systems, a feedback-linearized controller coupled with a Kalman filter has been described [11]. Performance limitations that degraded tracking accuracy suggested the use of a more robust controller design approach, such as sliding mode. A fast-tool servo based on MSL is depicted in Fig. 2. This type of actuator can be used in high-precision machining operations such as diamond turning of nonrotationally symmetric surfaces.

To use (10) to define a control action for the MSL system, it will be first assumed that it is possible to represent the magnetic forces as

$$\begin{aligned}F_1&=b_{11}h_1(i_1,x)=b_{11}\frac{i_1^2}{(x_1+x)^2},\\ F_2&=b_{22}h_2(i_2,x)=b_{22}\frac{i_2^2}{(x_2-x)^2}\end{aligned} \tag{16}$$

where F_1 is the force exerted by the magnets on one side, F_2 is the corresponding force in the opposite side, x_1 and x_2 are the equilibrium gaps at each side, x is the actuator position as defined in Fig. 1, and $b_{11}<0$ and $b_{22}>0$ are uncertain parameters with known bounds. It is possible to express the state equations of a magnetically levitated system in a way that the

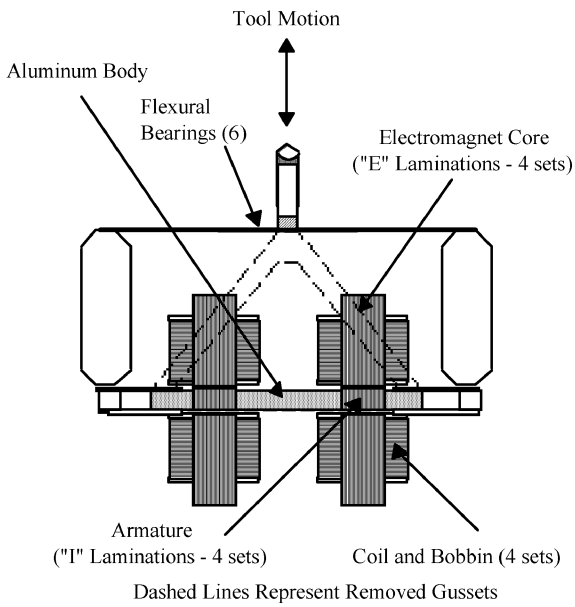

Fig. 2. Magnetically servo-levitated fast-tool servo.

control action is coil voltage instead of coil current [7], [14]. This approach has the appeal of providing system equations that are affine on the input, but its performance is severely hindered by the accumulated modeling error incurred when adding the equations that express the electromagnetic relationship between coil voltage and magnetic force. The simplifications involved severely deteriorate closed-loop performance and are unsuitable for high-precision fast tracking, as has been shown in previous attempts of controlling the system in the voltage mode [15].

The MSL actuator was first treated as a single-input-single-output (SISO) system with only one pair of magnets active at any given time, depending on the sign of the tracking error. In this case, $D = 1$ and, hence, $S = S^*$. From this and the modified sliding condition (10), the following control algorithm is proposed:

$$i_1 = \sqrt{\frac{-n(\vec{x})(x_1 + x)^2}{\max b_{11}}},\ i_2 = 0, \qquad \text{if } S > 0$$
$$i_2 = \sqrt{\frac{n(\vec{x})(x_2 - x)^2}{\min b_{22}}},\ i_1 = 0, \qquad \text{if } S < 0 \qquad (17)$$

where $n(\vec{x})$ is some scalar function of the state vector such that $0 < |\rho_j(\vec{x})| < n(\vec{x})$, $j = 1, 2$. To show how (17) meets the modified sliding condition, two cases need to be analyzed. For $S > 0$, the current i_1 must be squared and the resulting equation solved for $n(\vec{x})$. The corresponding expression represents a lower bound for the magnetic force $F_1 = b_{11}h_1(i_1, x)$. Since $n(\vec{x})$ is an upper bound for $|\rho(\vec{x})|$, it can be established that $-h_1(i_1, x) > \rho(\vec{x})$ and the sliding condition is thus satisfied. For $S < 0$, the procedure is similar.

Equation (17) has been simulated to evaluate its performance. Bounds for b_{11} and b_{22} were obtained from recorded input–output data corresponding to exciting the open-loop system with band-limited noise. Tracking performance and the corresponding command currents are shown in Fig. 3; the sliding surface corresponded to (6) and full state feedback was assumed (both position and velocity available). Defining the sliding surface as in (3) redefines $\partial S/\partial \vec{y}$, $\partial S/\partial t$ and $\rho(\vec{x})$, with control command still given by (17). The resulting algorithm was also simulated, and tracking response and corresponding command currents are shown in Fig. 4, where full state feedback was again assumed. Both controllers described in this section incorporate a boundary layer [2], [3], [5] to reduce control chatter. If the entire trajectory lies within the boundary layer, both magnets are active at all times, as seen in Figs. 3 and 4. This represents a tradeoff between the demand placed on the power amplifiers due to fast switching and the electric power dissipated on the actuator as heat.

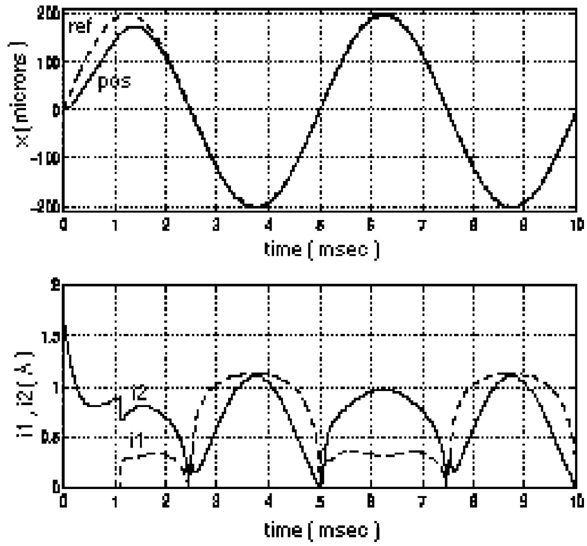

Fig. 3. Simulated tracking performance (17); 400 μm sine wave at 200 Hz.

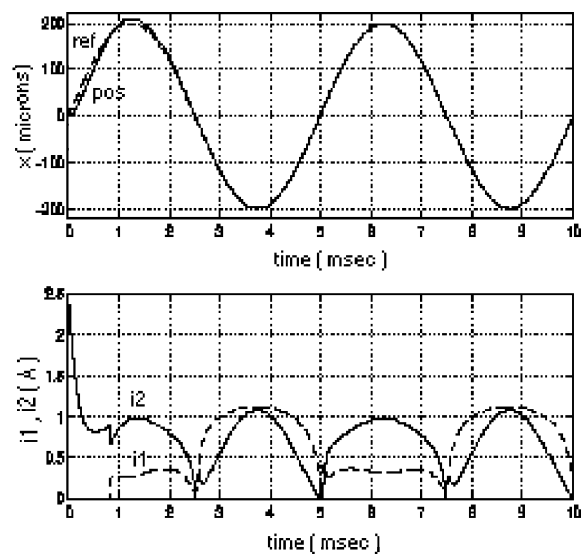

Fig. 4. Simulated tracking performance (16) using sliding surface (3); 400 μm sine wave at 200 Hz.

Although perfect tracking can be theoretically achieved by this design, several implementation issues degrade its potential performance. Equation (17) produces a highly chattered control action, which can worsen due to process noise, model uncertainties, parameter variations, performance of the state estimator, computational, and conversion delays. A tradeoff between control chatter and tracking accuracy must eventually be established.

The magnetically levitated fast-tool servo is shown in Fig. 5. The proposed algorithm [(17)] was implemented in a single-

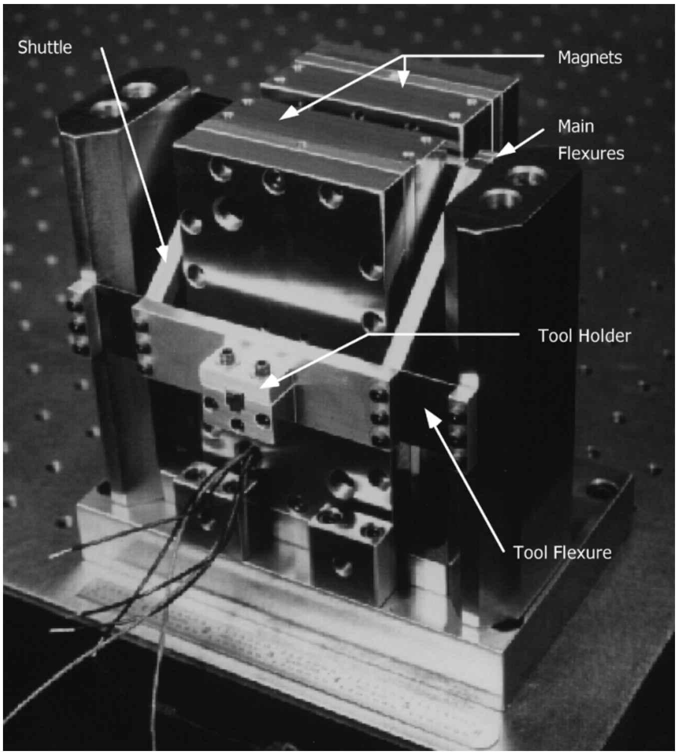

Fig. 5. Magnetically levitated fast-tool servo.

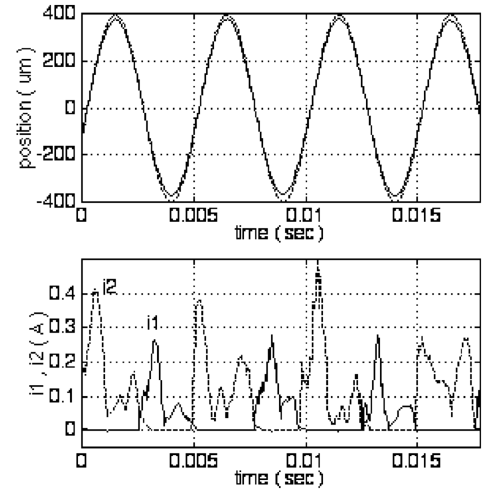

Fig. 6. Measured tracking performance (17). Sliding surface given by (3).

board DSP computer, with a laser interferometer as feedback sensor. Measured tracking performance and the corresponding coil currents are shown in Fig. 6, where reference command is shown as the dotted line and the measured response as the solid line (top plot). This controller was able to achieve fast tracking over a large range of motion with near-zero phase lag, providing performance independent of frequency, i.e., no phase degradation at higher speeds, as would be expected from a linear controller. It provided significantly better performance than the previously reported feedback linearization scheme [11] in spite of being based on a very simple force model [(16)].

An equivalent control term (continuous control action that drives the system once sliding has been achieved) can be included in the algorithm at the expense of increasing control signal power. Such a term is expected to improve tracking accuracy since the system driven by a purely switching control law oscillates around the sliding surface.

V. Modified Sliding Control Including Equivalent Control

Sliding-mode control comprises two synergistic actions: reaching the sliding surface and letting the state vector ride on it once sliding has been achieved. These are achieved by two different components of the control action: a switching term (u_{sw}) and the equivalent control (u_{eq}). The robustness of sliding control is given by the switching term u_{sw} derived from the sliding condition [in the nonaffine case, (10)]. This term takes the system from an arbitrary set of initial conditions to the sliding surface and accommodates parametric uncertainties and unmodeled dynamics by virtue of the switching action [3]–[5]. Once sliding has been achieved, the system dynamics should match that of the sliding surface. Equivalent control (u_{eq}) is the control action that drives the system once sliding has been achieved and is calculated from solving $dS/dt = 0$, hence being a continuous function [5]. It is a nonrobust control law, since it is based on the plant equations. The equivalent control is a convex combination of the values of u on both sides of the sliding surface, and its formal justification was derived by Filippov [19]. The robustness of sliding-mode control, however, lies on the switching portion of the control law (i.e., the one that meets the sliding condition), and, hence, the equivalent control action can be seen as the limiting behavior of the system's dynamics when switching occurs infinitely fast [3].

In affine systems, both actions can be combined into a single control law ($u = u_{\text{eq}} + u_{sw}$), and several switching algorithms can be derived by forcing the total control action u to satisfy the sliding condition. In this section, an extension of these ideas to the nonaffine case is presented. A purely switching controller [such as (17)] works because it satisfies the modified sliding condition, but it can only provide high performance at the expense of very fast switching (chatter). The inclusion of an equivalent control term would improve tracking performance while reducing chatter, alleviating the performance tradeoff involved in the use of a boundary layer. Developing an equivalent control term to be combined with the switching action relies heavily on the type of input nonlinearity. The proposed concept will be illustrated by adding an equivalent control term to the previously discussed MSL controller.

To calculate the equivalent control term, the MSL actuator will first be treated as a SISO system with only one pair of magnets active at any given time. The sliding surface S becomes one-dimensional; then $dS/dt = 0$ yields

$$\dot{S} = \frac{\partial S}{\partial t} + \frac{\partial S}{\partial \vec{y}}\dot{y} = \frac{\partial S}{\partial t} + \frac{\partial S}{\partial \vec{y}}\left(\begin{bmatrix} f_1(\vec{x}) \\ f_2(\vec{x}) \end{bmatrix} + \begin{bmatrix} 0 \\ F_{\text{eq}} \end{bmatrix}\right) = 0 \tag{18}$$

and, from (3), $\partial S/\partial \vec{y} = [2\lambda \; 1]$. Equation (18) can be solved for the equivalent control input (force F_{eq})

$$F_{\text{eq}} = -\frac{\partial S}{\partial t} - 2\lambda f_1 - f_2 = b\frac{i_{\text{eq}}^2}{(x_0 \pm x)^2} \tag{19}$$

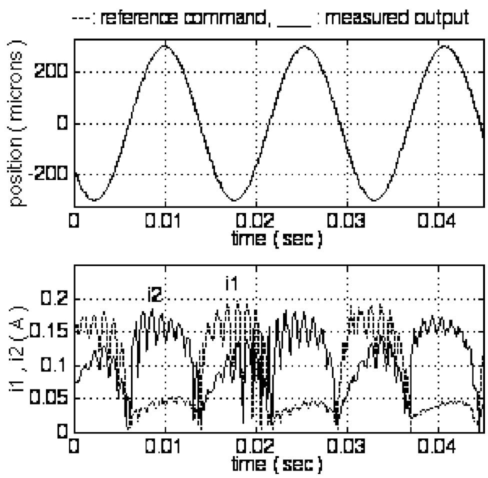

Fig. 7. Measured tracking performance, including equivalent term; 600 μm travel.

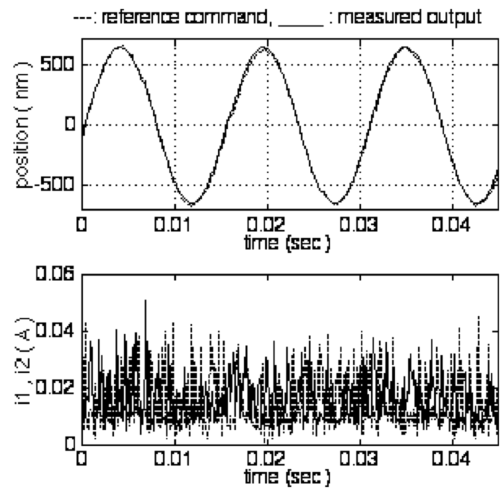

Fig. 8. Measured tracking performance, including equivalent term; 1.2 μm travel.

which in turn can be solved for i_{eq}

$$i_{\mathrm{eq}} = (x_0 \pm x)\sqrt{\frac{\left(-\frac{\partial S}{\partial t} - 2\lambda f_1 - f_2\right)}{b}}. \tag{20}$$

An equivalent control action can now be proposed. For instance, for the left coil, we have

$$i_{\mathrm{eq}_1} = \begin{cases} (x_1+x)\sqrt{\frac{\left(-\frac{\partial S}{\partial t}-2\lambda f_1-f_2\right)}{b_{11}}}, & \text{if} -\frac{\partial S}{\partial t}-2\lambda f_1-f_2 < 0 \\ 0, & \text{otherwise} \end{cases}. \tag{21}$$

To prove that the total control action $i = i_{sw} + i_{\mathrm{eq}}$, where i_{sw} is given by (17), meets the modified sliding condition [(10)], four cases need to be analyzed (two per magnet). For the left coil, consider first $S > 0$ and $-\partial S/\partial t - 2\lambda f_1 - f_2 < 0$. After some algebra, the total control action becomes twice the current i_1 in (17); i_1 can then be squared and the resulting equation solved for $n(\vec{x})$. The resulting expression, which corresponds to a quarter of the force exerted by the magnet, represents a lower bound for the magnetic force $F_1 = b_{11}h_1(i_1, x)$. Since $n(\vec{x})$ is an upper bound for $|\rho(\vec{x})|$, it can be established that $-h_1(i_1, x) > \rho(\vec{x})$, and the sliding condition is thus satisfied. For the case where $-\partial S/\partial t - 2\lambda f_1 - f_2 > 0$, $i_{\mathrm{eq}} = 0$ and the total control action becomes the same as the one given by (17), which meets the modified sliding condition as has been shown before. The proof for the other magnet is identical. A sliding controller combining (17) and (21) was implemented and tested. Results from tracking experiments are plotted in Figs. 7 and 8, showing that the reference command and measured response overlap. Near-zero phase lag and very low harmonic distortion have been achieved by this algorithm.

High-resolution tracking can be achieved by using thinner boundary layers, with the corresponding chattered control currents. The use of an equivalent control term, however, significantly improves tracking accuracy for a fixed value of boundary layer thickness.

VI. Conclusion

A novel control algorithm for highly accurate, wide-band control of MSL has been presented. The proposed technique, which is an extension of some basic concepts from the standard sliding-mode formulation to nonaffine nonlinear systems with algebraic input invertibility, has shown excellent tracking performance over a long range of motion at high speeds in an MSL-based fast-tool servo system for diamond turning. Submicrometer accuracy over a long range of motion can be achieved, with near-zero phase lag, moderate chatter, and low power consumption.

Sliding-mode control of nonaffine systems can be approached as a two-step design. First, a control law that meets the modified sliding condition (10) is devised. Typically, this is a switching control law that drives the system from an arbitrary set of initial conditions to sliding, and provides robustness to parametric uncertainty. Second, an equivalent control term can be added. The equivalent control term is calculated by solving for u in $dS/dt = 0$, and the combination of both equivalent and switching terms has to be such that the total control action satisfies the modified sliding condition. While the proposed controller can theoretically provide perfect tracking within certain bounds of parametric uncertainty, implementation issues such as control chatter, speed estimation, and computational time delay limit the ideal performance. Sliding mode assumes full state feedback, and therefore an observer or some other estimation technique could be necessary. Chatter can limit the performance of systems controlled by sliding mode since actuators can only provide finite slew rates, making the implementation of purely switching algorithms unfeasible. Chatter can be alleviated by using a boundary layer, reducing fast switching at the expense of increasing tracking error. A tradeoff between actuator chatter, tracking accuracy, and rms control effort must eventually be established. The use of an equivalent control term significantly improves tracking accuracy without increasing chatter, as the use of a thinner boundary layer would do. The equivalent control term depends on the nature of the input nonlinearity, and its combination with the switching term is not straightforward since the effect of both terms (u_{sw} and u_{eq}) in the nonaffine case cannot be analyzed by superposition. Proposing an equivalent control term requires some physical insight into the nature of the nonlinearity, and the combined control action must be tested analytically to show that the global control action meets the modified sliding condition.

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Hector Gutierrez (M'97) was born in Lima, Peru. He received the B.Sc. degree in applied mathematics from the Universidad Cayetano Heredia, Lima, Peru, in 1989, the B.Sc. degree in mechanical engineering from the Pontificia Universidad Catolica, Lima, Peru, in 1991, and the M.Eng. degree in manufacturing systems and the Ph.D. degree in electrical engineering from North Carolina State University, Raleigh, in 1993 and 1997, respectively.

Since August 1999, he has been with the Department of Mechanical and Aerospace Engineering, Florida Institute of Technology, Melbourne, where he is currently an Associate Professor. His professional interests are in magnetic suspension systems (both electromagnetic and electrodynamic), electromechanical energy conversion, electromagnetic launching systems, mechatronics, and nonlinear control.

Paul I. Ro was born in Seoul, Korea, in 1959. He received the B.S. degree from the University of Minnesota, Minneapolis, in 1982, and the M.S. and Ph.D. degrees from the Massachusetts Institute of Technology, Cambridge, in 1985 and 1989, respectively, all in mechanical engineering.

He is currently a Professor in the Mechanical and Aerospace Engineering Department, North Carolina State University, Raleigh. His fields of research are precision machining, magnetic servo levitation, nonholonomic motion planning and control, robotic assembly, and intelligent vehicle control.

Dr. Ro is a Member of the American Society of Mechanical Engineers and the American Society of Precision Engineers. He is an Associate Editor of *Precision Engineering*.