1. Write your answers clearly. To get complete credit, adequate work must be shown to support your answers.

2. The numbers on the right indicate the maximum credit for the corresponding question.

3. Use of calculators is NOT permitted.

1. Find all values of \( a, b, c \) for which \( A \) is symmetric.

\[
\begin{pmatrix}
2 & a-2b+2c & 2a+b+c \\
3 & 5 & a+c \\
0 & -2 & 7
\end{pmatrix}.
\] [10]

2. For which real values of \( \lambda \) do the vectors \( v_1 = (\lambda, \frac{3}{2}, \frac{1}{2}) \), \( v_2 = (\frac{1}{2}, \lambda, \frac{1}{2}) \), \( v_3 = (\frac{1}{2}, \frac{1}{2}, \lambda) \) form a linearly dependent set in \( \mathbb{R}^3 \).

[8]

3. Factor the matrix \( A = \begin{pmatrix}
0 & 1 & 7 & 8 \\
1 & 3 & 3 & 8 \\
-2 & -5 & 1 & -8
\end{pmatrix} \) as \( A = EFGR \) where \( E, F, G \) are elementary matrices and \( R \) is in row echelon form.

[10]

4. Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given row echelon form. Solve the system by reducing the matrix to reduced row echelon form. Assume that the variables are named \( x_1, x_2, ..., \) from left to right.

\[
\begin{pmatrix}
1 & 7 & -2 & 0 & -8 & -3 \\
0 & 0 & 1 & 1 & 6 & 5 \\
0 & 0 & 0 & 1 & 3 & 9 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}.
\] [8]

5. If \( A \) is a matrix with \( n \) columns, then prove that the solution space of the homogeneous system \( Ax = 0 \) if all of \( \mathbb{R}^n \) if and only if \( A = 0 \). [6]

6. (a) Find a homogeneous linear system of two equations in three unknowns whose solution space consists of vectors that are orthogonal to \( a = (1, 1, 1), b = (-2, 3, 0) \). [4]

(b) Solve the system obtained in (a). [4]