1. Let \( A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & 1 \\ -4 & 1 & 2 \end{bmatrix} \) and \( b = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} \).

Express the product \( Ab \) as a linear combination of the columns of \( A \). And find the product \( Ab \).

2. Multiply \( A \) times \( x \) to find the components of \( Ax \).

\[ A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}. \]

3. What \( 3 \times 3 \) matrix \( E \) multiplies \( \begin{bmatrix} x \\ y \\ z \end{bmatrix} \) to give \( \begin{bmatrix} x \\ y \\ z + x \end{bmatrix} \).

4. What \( 2 \times 2 \) matrix \( P_1 \) projects the vector \( (x, y) \) onto \( x \) axis to produce \( (x, 0) \)?

What \( 2 \times 2 \) matrix \( P_1 \) projects the vector \( (x, y) \) onto \( y \) axis to produce \( (0, y) \)?

5. Choose a RHS which gives no solution and another RHS that gives infinitely many solutions. Find at least two of those solutions.

\[ \begin{align*}
3x + 2y &= 7 \\
6x + 4y &= ?
\end{align*} \]

6. Interpret the system of equations

\[ \begin{align*}
3x - 2y &= 7 \\
6x - 4y &= 6
\end{align*} \]

in terms of linear combinations of vectors. You don’t have to solve the system!

7. Write down \( 3 \times 3 \) matrices that produce these elimination steps:

(i) \( E_{21} \) subtracts 5 times row 1 from row 2.
(ii) \( E_{32} \) subtracts -7 times row 2 from row 3.
(i) \( P_{12} \) exchanges rows 1 and 2.

8. Find three elimination (elementary) matrices \( E_1, E_2, E_3 \) such that

\[ E_3E_2E_1A = U \text{ where } A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} \text{ and } U \text{ is an upper triangular matrix.} \]

Multiply those \( E' \)s to get one matrix \( M \) such that \( MA = U \).
9. Explain these facts. If the third column of a matrix $B$ is all zero, then the third column of $EB$ is all zero, where $E, B$ are any two matrices such that their product is defined. If third row of $B$ is all zero, the third row of $EB$ might not be zero.

10. Write down the $3 \times 3$ matrix that has $a_{ij} = 2i - 3j$. Transform this matrix to an upper triangular form.

11. Multiply the matrices in the orders $EF$ and $FE$. $E = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{bmatrix}$, $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. Compute $E^2, F^2$.

12. Show that $(A + B)^2$ is different from $A^2 + 2AB + B^2$, when $A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}$.

13. Consider the lower triangular matrix $L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$. Find $L^{-1}$ by Gauss Jordon method and show that $L^{-1}$ is also lower triangular.