1. Precise and complete answers are a must for full credit.

2. Show all your work. Calculators are NOT allowed.

1. Use Gram-Schmidt process to transform the basis \( B = \{(1, 1, 1), (-1, 1, 0), (1, 2, 1)\} \) of \( \mathbb{R}^3 \) into an orthonormal basis. \( [10] \)

2. Find the eigenvalues of \( A = \begin{bmatrix} 1 & 1 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \) and their algebraic and geometric multiplicities. \( [12] \)

3. Find a matrix \( P \) that diagonalizes \( A = \begin{bmatrix} -14 & 12 \\ -20 & 17 \end{bmatrix} \) and find a matrix \( D \) that is similar to \( A \). \( [10] \)

4. Let \( A = \begin{bmatrix} 4 & 16 & 8 & 8 \\ 0 & 0 & 5 & 15 \\ 0 & 0 & 4 & 9 \\ 1 & 4 & 2 & 2 \end{bmatrix} \); \( R = \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \); \( C = \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{3}{5} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \).

The reduced row echelon form of \( A \) is \( R \) and the reduced row echelon form of \( A^T \) is \( C \).

Find a basis for each of the following spaces: \( [5+5+4+4] \)

(a) column space of \( A \)
(b) row space of \( A \) consisting of the row vectors of \( A \)
(c) null space of \( A \)
(d) null space of \( A^T \).

5. Determine whether the linear operator \( T : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) defined by the equations

\[
\begin{align*}
w_1 &= x_1 + 2x_2 \\
w_2 &= x_1 + x_2
\end{align*}
\]

is one-one. If it is one-one, find the inverse operator \( T^{-1} \). \( [8] \)
6. Let \( T : \mathbb{R}^n \to \mathbb{R}^n \) be a linear transformation. Prove that \( T \) is one-one if and only if \( \ker(T) = \{0\} \). [8]

7. Confirm that the matrix \( A = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{2}{\sqrt{6}} \end{bmatrix} \) is orthogonal and find its inverse. [8]

8. Find the determinant of the matrix

\[
\begin{bmatrix}
a & b & b & b \\
b & a & b & b \\
b & b & a & b \\
b & b & b & a
\end{bmatrix}
\] . [8]

9. Find a \( LDU \)-decomposition of \( A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & 0 & 2 \\ 2 & 2 & 1 \end{bmatrix} \). [10]

10. Use matrix multiplication to find the image of the point \( (4,3) \) under the transformation of reflection about the line through origin that makes an angle of \( \theta = \frac{\pi}{4} \) with the positive \( x \) axis. [8]

Education is for Life not just for a living.