Practice Problems

MTH 3102 1/26/2008

1. Let $A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 5 \\ -2 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 8 & -3 & -5 \\ 0 & 1 & 2 \\ 4 & -7 & 6 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & -2 & 3 \\ 1 & 7 & 4 \\ 3 & 5 & 9 \end{bmatrix}$.

   (a) Find a matrix $X$ that satisfies the equation, $B + (A + X)^T = C$.

   (b) Find a matrix that satisfies the equation $B + (\text{tr}(A)X)^T = C$.

2. A square matrix $A$ is said to be symmetric if $A = A^t$. That is $a_{ij} = a_{ji}$ for $1 \leq i, j \leq n$. Show that for every square matrix $A$, the matrix $B = A + A^t$ is symmetric. Is every symmetric matrix of this form?

3. Suppose $A$ is a symmetric $n \times n$ matrix. Show that $AA^t$ and $A^tA$ are symmetric.

4. Show, by giving an example, that for square matrices, $(A + B)(A - B)$ need not be equal to $A^2 - B^2$.

5. Prove that for any scalar $2 \times 2$ matrix $A$ there are scalars $c_1, c_2, c_3, c_4$ such that the linear combination $c_1I + c_2A + c_3A^2 + c_4A^3 = 0$ where $0$ is the zero matrix and $I$ is the identity matrix.