1. Determine whether the given functions are continuous. Explain clearly.

(a) \( f(x) = \frac{|x|}{x} \), where \( x = \pm \frac{1}{n}, \ n \in \mathbb{N} \)

(b) \( f(x) = \begin{cases} x \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \)

(c) \( f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \)

(d) \( f(x) = \begin{cases} e^{(-\frac{1}{x^2})} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \)

2. Determine if the functions given below are continuous or not at the given points.

(i) \( f(x) = \begin{cases} x & \text{if } x = \frac{1}{n} \text{ and } n \in \mathbb{Z}\setminus\{0\} \\ 1 - x & \text{otherwise} \end{cases} \)

(ii) \( f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 1 - x & \text{if } x \text{ is irrational} \end{cases} \)

3. Suppose that a function \( f \) is continuous on \((a, b)\) and \( f(r) = c \) for all rational \( r \) in \((a, b)\) and \( c \) is some fixed real constant. Prove that \( f(x) = c \) for all \( x \in (a, b) \).

4. Give examples of the following type of functions:

(i) function \( f \) defined on \( \mathbb{R} \) but not continuous at any point of \( \mathbb{R} \)

(ii) function \( f \) defined on \( \mathbb{R} \) but continuous at exactly one point of \( \mathbb{R} \)

(iii) function \( f \) defined on \([a, b]\) but continuous only at denumerably many points of \([a, b]\).

5. For each of the given functions, locate and classify all the points of discontinuity.

(i) \( f(x) = \frac{|x|}{x}, \ \text{where } x = \pm \frac{1}{n}, \ n \in \mathbb{N} \)

(ii) \( f(x) = \frac{|x| + x}{2}, \ x \in (-\frac{3}{2}, 1] \)