(1) Confirm that the vectors \( \mathbf{v}_1 = (2/3, 1/3, 2/3) \), \( \mathbf{v}_2 = (1/3, 2, -1/3) \),
\( \mathbf{v}_3 = (1/3, -2/3, -1/3) \) is an orthonormal basis for \( \mathbb{R}^3 \) and express
\( \mathbf{w} = (2, 0, 5) \) as a linear combination of those vectors.

(2) Confirm that the vectors \( \mathbf{v}_1 = (1, 1, 1, 2) \), \( \mathbf{v}_2 = (1, 2, 3, -3) \),
\( \mathbf{v}_3 = (1, -2, 1, 0) \) and \( \mathbf{v}_4 = (25, 4, -17, 6) \) is an orthonormal
basis for \( \mathbb{R}^4 \) and express \( \mathbf{w} = (1, 1, 1, 1) \) as a linear
combination of those vectors.

(3) Use Gram-Schmidt process to transform the given basis into
an orthonormal basis:
\( (i) \) \{\( (1, -3), (2, 2) \)\} \( (ii) \) \{\( (1, 1, 1), (-1, 1, 0), 1, 2, 1) \}\}
\( (iii) \) \{\( (0, 2, 1, 0), (1, -1, 0, 0), (1, 2, 0, -1), (1, 0, 0, 1) \}\}

(4) Extend the given orthonormal set to an orthonormal basis for
\( \mathbb{R}^3 \) or \( \mathbb{R}^4 \) (as appropriate) by finding vectors that are
orthogonal to the given vectors.
\( (i) \) \( \mathbf{v}_1 = (2/3, 2, 0, 1/3) \), \( \mathbf{v}_2 = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}, 0) \).
\( (ii) \) \( \mathbf{v}_1 = (1/\sqrt{2}, 1/\sqrt{2}, 0) \), \( \mathbf{v}_2 = (1/\sqrt{2}, 1/\sqrt{2}, 0) \).

(5) Find an orthonormal basis for the subspace of \( \mathbb{R}^3 \) spanned by
the vectors \( \mathbf{v}_1 = (0, 1, 2) \), \( \mathbf{v}_2 = (-1, 0, 1) \), \( \mathbf{v}_3 = (-1, 1, 3) \).

(6) Find an orthonormal basis for the subspace of \( \mathbb{R}^4 \) spanned by
the vectors \( \mathbf{v}_1 = (-1, 2, 4, 7) \), \( \mathbf{v}_2 = (-3, 0, 4, -2) \),
\( \mathbf{v}_3 = (2, 7, -3) \), \( \mathbf{v}_4 = (4, 4, 7, 6) \).