1. Find the eigenvalues and eigen vectors of
   i) \( A = \begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix} \). Solution: (i) \( \lambda = 4, 4; X = t \begin{bmatrix} 3 \\ 2 \end{bmatrix} \)
   
   (ii) \( A = \begin{bmatrix} 3 & 4 & -1 \\ -1 & -2 & 1 \\ 3 & 9 & 0 \end{bmatrix} \)
   
   Solution: (i) \( \lambda_1 = 2, 2; X = t \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}; \lambda_2 = -3; X = t \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} \).

2. Find the eigen values and eigen vectors of \( A \) and of the stated power of \( A \).
   (i) \( A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}; \quad A^{25} \)
   
   Solution: \( \lambda_1 = -1; X = t \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}; \lambda_2 = 1; X = t \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} + s \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \).
   
   The eigen values of \( A^{25} \) are \( \lambda = (-1)^{25} = -1 \) and \( \lambda_2 = (1)^{25} = 1. \)

3. For what value(s) of \( x \) if any does the matrix \( A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & x & 2 \\ 0 & 2 & x \end{bmatrix} \), has atleast one repeated eigenvalue. (solution: \( x = 1 \) or \( x = 5 \).)

4. Let \( A \) be a \((2 \times 2)\) matrix such that \( A^2 = I \). For any \( x \in \mathbb{R}^2 \), if \( x + Ax \) and \( x - Ax \) are eigenvectors of \( A \) find the corresponding eigenvalue.

5. Prove that if \( A \) is a square matrix then \( A \) and \( A^T \) have the same characteristic polynomial.

6. Let \( A = \begin{bmatrix} 2 & 0 \\ 2 & 3 \end{bmatrix} \). Show that \( A \) and \( A^T \) do not have the same eigen spaces.

7. If \( \lambda \) is an eigen value of \( A \) and \( X \) is the corresponding eigenvector, then prove that \( \lambda - s \) is an eigen value of \( A - sI \) for any scalar \( s \) and \( X \) is the corresponding eigenvector.

8. If \( \lambda \) is an eigen value of an invertible matrix, and \( x \) is a corresponding eigen vector, then show that \( \frac{1}{\lambda} \) is an eigen value of \( A^{-1} \) and \( x \) is a corresponding eigen vector.
9. Find eigen values and eigen vectors of the following matrices:

(i) \[
\begin{bmatrix}
4 & -5 \\
1 & 0
\end{bmatrix}
\]. (ii) \[
\begin{bmatrix}
5 & -2 \\
1 & 3
\end{bmatrix}
\].

10. Find the eigen values of the matrix \( A = \begin{bmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{bmatrix} \). If \( x \) is an eigen vector corresponding to either of the eigen values of \( A \), then show that \( \text{Re}(x) \) and \( \text{Im}(x) \) are orthogonal and have the same length.