I Mid Term Examination

MTH 3102 9/24/2008

Max. Points: 50 Time: 60 minutes

Write your answers clearly. To get complete credit, adequate work must be shown to support your answers. The numbers on the right indicate the maximum credit for the corresponding question. Use of calculators is NOT permitted.

1. Let \( A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix} \). [6 + 6 + 6]

   (a) Use the properties of determinants and evaluate the determinant of \( A \).
   (b) Find the adjoint of \( A \), and the inverse of \( A \), if it exists.
   (c) Use the \( A^{-1} \) obtained above and find the polynomial of degree 2, that passes through \((-1, 0), (1, 4) \) and \((2, 3)\).

2. Find the LDU decomosition of the matrix \( A = \begin{bmatrix} 3 & -6 \\ -2 & 5 \end{bmatrix} \). [6]

3. If \( A \) is an arbitrary skew symmetric \((2n - 1) \times (2n - 1)\) matrix, show that \( \det (A) = 0 \). [4]

4. Factor the matrix \( A = \begin{bmatrix} 0 & 1 & 7 & 8 \\ 1 & 3 & 3 & 8 \\ -2 & -5 & 1 & -8 \end{bmatrix} \) as \( A = EFGR \) where \( E, F, G \) are elementary matrices and \( R \) is in row echelon form. [8]

5. If \((I + 2A)^{-1} = \begin{bmatrix} -1 & 2 \\ 4 & 5 \end{bmatrix}\) find \( A \). [4]

6. Give examples for any five of the following: (Wherever necessary, show your work to support that your examples are valid.) [10]

   (a) A \((3 \times 3)\) symmetric matrix with trace zero.
   (b) Two vectors in \( \mathbb{R}^4 \) that are orthonormal.
   (c) Two square matrices \( A \) and \( B \) such that \((A + B)(A - B) \neq A^2 - B^2\).
   (d) A \(2 \times 2\) matrix \( A \) that is idempotent.
   (e) A \(2 \times 2\) matrix \( A \) that is nilpotent matrix of index 2.
   (f) Two nonzero matrices \( A \) and \( B \) such that the product \( AB = 0 \), where \( 0 \) is the zero matrix.