1. Let $V = \mathbb{R}^2$ and $W_1 = \{(a_1, 0) : a_1 \in \mathbb{R}\}$. Give examples of two different subspaces $W_2, W_2^*$ such that $V = W_1 \oplus W_2$ and $V = W_1 \oplus W_2^*$. [4]

2. Prove that if $W_1$ and $W_2$ are finite dimensional subspaces of a vector space $V$ then the subspace $W_1 + W_2$ is also finite dimensional. And show that $\dim (W_1 + W_2) = \dim(W_1) + \dim (W_2) - \dim (W_1 \cap W_2)$. [4]

3. Let $T : M_{2 \times 2}(\mathbb{R}) \to \mathbb{R}$ defined by $T(A) = tr(A)$. Find nullity $T$ and rank $T$. Verify the dimension theorem. [4]

4. Let $T : V \to W$. Prove that $T$ is one to one if and only if $T$ carries linearly independent subsets of $V$ onto linearly independent subsets of $W$. [4]

5. Let $T : V \to W$ be vector spaces of equal (finite) dimension. Let $T : V \to W$ be linear. Prove that $T$ is one-to-one if and only if $T$ is onto. Use this result to determine if $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3, a_2)$ is onto? [4]