Linear Algebra : Spring 2006  
Mid Term Examination I

MTH 5102  
2/1/2006  

Max Credit : 20 points  
Duration : 60 minutes

1. Answer all the questions. The numbers on the right in brackets indicate the maximum credit the corresponding question carries.

2. Precise and complete answers are a must for full credit.

In this question paper, the symbol $V$ stands for an arbitrary vector space over a field $F$.

1. Let $\mathbb{R}^2 = \{(a_1, a_2) | a_1, a_2 \in \mathbb{R}\}$. Define addition of elements of $\mathbb{R}^2$ coordinatewise, and for $(a_1, a_2) \in \mathbb{R}^2$ and $c \in \mathbb{R}$ define the multiplication by scalars as:

$$c(a_1, a_2) = \begin{cases} (0, 0) & \text{if } c = 0 \\ (ca_1, ca_2) & \text{if } c \neq 0. \end{cases}$$

Is $\mathbb{R}^2$ a vector space with these operations? Justify your answer. [4]

2. Let $W$ be a subspace of $V$. For $v_1, v_2 \in V$, prove that $v_1 + W = v_2 + W$ if and only if $(v_1 - v_2) \in W$. [4]

3. Let $u, v, w$ be distinct vectors in $V$. Prove that the set $\{u, v, w\}$ is linearly independent if and only if the set $\{u + v, u + w, v + w\}$ is linearly independent. [4]

4. Let $W_1, W_2$ be subspaces of $V$. Prove that $W_1 \cup W_2$ is a subspace of $V$ if and only if either $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$. [4]

5. Give an example for each of the following sets, in any vector space $V$ of your choice. Justify with reasons as to why your examples are appropriate. [4]

(a) A generating set of $V$ that is linearly dependent.
(b) A linearly independent set that is not a generating set of $V$.
(c) A subset that is not a subspace of $V$.
(d) A subset $S$ and the smallest subspace containing $S$. And also, the largest subspace containing $S$. 

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