1. Let $\beta$ and $\gamma$ be the standard bases for $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively. For each linear transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ given below, compute $[T]_\beta$.
   (i) $T(a_1, a_2, a_3) = (2a_1 + 3a_2 - a_3, a_1 + a_3)$.
   (ii) $T(a_1, a_2, a_3) = 2a_1 + a_2 - 3a_3$.

2. Define $T : M_{2 \times 2}(\mathbb{R}) \rightarrow P_2(\mathbb{R})$ by
   $$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a + b) + 2dx + bx^2.$$ 
   Let $\beta = \{(1,0,0), (0,1,0), (0,0,1), (0,0,0)\}$ and $\gamma = \{1, x, x^2\}$.
   Compute $[T]_\beta$.

3. Let $V$ be an $n$-dimensional vector space with an ordered basis $\beta$. Define $T : V \rightarrow F$ by $T = [x]_\beta$. Prove that $T$ is linear.

4. Let $V$ be a vector space with the ordered basis $\beta = \{v_1, v_2, \ldots, v_n\}$. Define $v_0 = 0$. Let $T$ be a linear transformation such that $T(v_j) = v_j + v_{j-1}$, for $j = 1, 2, \ldots, n$. Compute $[T]_\beta$.

5. Let $V$ be a finite dimensional vector space and $T$ be the projection on $W$ along $W'$, where $W$ and $W'$ are subspaces of $V$. Find an ordered basis $\beta$ for $V$ such that $[T]_\beta$ is a diagonal matrix.

6. Let $g(x) = 3 + x$. Let $T : P_2(\mathbb{R}) \rightarrow P_2(\mathbb{R})$ and $U : P_2(\mathbb{R}) \rightarrow \mathbb{R}^3$ be the linear transformations respectively defined by
   $$T(f(x)) = f'(x)g(x) + 2f(x) \quad U(a + bx + cx^2) = (a, b, c, a - b).$$ 
   Let $\beta$ and $\gamma$ be the standard ordered bases of $P_2(\mathbb{R})$ and $\mathbb{R}^3$, respectively.
   (i) Compute $[U]_\gamma^\beta$, $[T]_\beta^\gamma$ and $[UT]_\gamma^\beta$ directly.
   (ii) Let $h(x) = 3 - 2x + x^2$. Compute $[h(x)]_\beta$ and $[U(h(x))]_\gamma$. Use $[U]_\gamma^\beta$ and show that $[U(h(x))]_\gamma = [U]_\gamma^\beta[h(x)]_\beta$.

7. Use $[T]_\beta$ obtained in Q.2. above, and compute $[T(A)]_\gamma$ where $A = \begin{pmatrix} 1 & 4 \\ -1 & 6 \end{pmatrix}$.

8. Find linear transformations $U, T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ such that $UT = 0$, the zero transformation but $TU \neq 0$. Use your answer to find matrices $A$ and $B$ such that $AB = 0$ but $BA \neq 0$. 

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9. Let $V, W$ and $Z$ be vector spaces, and let $T : V \to W$ and $U : W \to Z$ be linear. Prove that (i) $UT$ is one-one then $T$ is one-one. (ii) If $UT$ is onto, then $U$ is onto.

10. Let $V$ be a finite dimensional vector space and let $T : V \to V$ be linear. If $\text{rank}(T) = \text{rank}(T^2)$, prove that $R(T) \cap N(T) = \{0\}$.