1. Find the LU decomposition of the matrix \( A = \begin{bmatrix} -2 & 2 & -4 & -6 \\ -3 & 6 & 3 & -15 \\ 5 & -8 & -1 & 17 \\ 1 & 1 & 11 & 7 \end{bmatrix}. \)

2. Use the LU decomposition obtained in Q.1 and find the determinant of \( A \) in Q.1. \([7+3]\)

3. Show that the matrix \( \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 8 & 1 & 0 \end{bmatrix} \) is nilpotent. Find \((I - A)^{-1}\), using the powers of \( A \). \([4+6]\)

4. Show that the vectors \( \mathbf{v}_1 = (0, 3, 1, -1), \mathbf{v}_2 = (6, 0, 5, 1), \mathbf{v}_3 = (4, -7, 1, 3) \) form a linearly dependent set in \( \mathbb{R}^4 \). \([6]\)

5. Find a general solution of the system, state the dimension of the solution space and confirm that the row vectors of the coefficient matrix \( A \) are orthogonal to the solution vectors.
\[
\begin{align*}
x_1 + 5x_2 + x_3 + 2x_4 - x_5 &= 0 \\
x_1 - 2x_2 - x_3 + 3x_4 + 2x_5 &= 0.
\end{align*}
\] \([6+4+6]\)

6. Find all values of \( x \) for which the matrix is invertible.
\[
(i) \begin{bmatrix} x - 1 & x_2 & +x_4 \\ 0 & x + 2 & x^3 \\ 0 & 0 & x - 4 \end{bmatrix}
\] \([8]\)