Homework 8: Linear Equations

1. Find the general solution of the given ODE and give the largest interval on which the solution is defined.
   (i) $y' + 3t^2y = t^2$.
   (ii) $t \frac{dy}{dt} - y = t^2 \sin t$.
   (ii) $t^2y' + t(t + 2)y = e^t$.
   (iv) $(\cos t) \frac{dy}{dt} + (\sin t)y = 1$.
   (v) $\frac{dr}{d\theta} + r \sec \theta = \cos \theta$

2. Find the solutions to the following IVPs:
   (i) $\frac{dT}{dt} = K(T - T_m)$, $T(0) = T_0$, $K, T_m, T_0$ are constants.
   (ii) $(t + 1) \frac{dx}{dt} + x = \ln t$, $x(1) = 10$.
   (iii) $\frac{dy}{dt} + y = f(t)$, $f(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ -1, & t > 1 \end{cases}$, $y(0) = 1$.

3. Solve $\frac{dy}{dt} - 2ty = 2$, $y(0) = 1$, in terms of $erf(t)$.

4. Solve $\frac{dy}{dt} - 2ty = -1$, $y(0) = \frac{\sqrt{\pi}}{2}$, in terms of $erfc(t)$.

5. Define $Si(t) = \int_0^t \frac{\sin u}{u} du$.
   Show that the solution of the IVP
   $t^3y' + 2t^2y = 10 \sin t$, $y(1) = 0$
   is $y(t) = 10t^{-2}[Si(t) - Si(1)]$.

6. Solve: $\frac{dy}{dt} - y = e^t y^2$.

7. Solve the IVP $y^k \frac{dy}{dt} + y^\frac{3}{2} = 1$, $y(0) = 4$. 

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