1. (a) Solve the following system of linear equations:

\[
\begin{align*}
\begin{align*}
a + b + c &= 0 \\
5a + 4b + c + d &= 0 \\
10a + 6b + d &= 2 \\
6a &= 1
\end{align*}
\end{align*}
\]

(b) Find the inverse Laplace transform of the following function:

\[
Y(s) = \frac{2s + 1}{s(s + 1)(s^2 + 4s + 6)}
\]

(c) Use Laplace transform methods to solve the following initial value problem:

\[
\begin{align*}
y'' + 4y' + 6y &= 1 + e^t \\
y(0) = 0, \ y'(0) &= 0
\end{align*}
\]

2. (a) Find a fundamental matrix \( \Phi(t) \) of the system of ODE

\[
\begin{align*}
x' &= 3x - y \\
y' &= 9x - 3y
\end{align*}
\]

(b) Find the inverse \( \Phi^{-1}(t) \) of the fundamental matrix \( \Phi(t) \) obtained in part (a).
(c) Find a particular solution \( \psi(t) \) of
\[
\begin{align*}
x'(t) &= 3x - y + 1 \\
y'(t) &= 9x - 3y + 3
\end{align*}
\]
using the formula
\[
\psi(t) = \Phi(t) \int_0^t \Phi^{-1}(s) f(s) \, ds
\]
where \( f = \left( \begin{array}{c} 1 \\ 3 \end{array} \right) \), and \( \Phi(t) \) is the fundamental matrix obtained in (a).

(d) Find the general solution of the nonhomogeneous system in (c).

3. (a) Transform the differential equation
\[
x^2y'' + xy' + 4y = \sin(2\ln x)
\]
into a differential equation with constant coefficients.

(b) Find the general solution of the differential equation with constant coefficients obtained in (a), using the method of undetermined coefficients.

(c) Use the solution obtained in (b) to find the general solution of the differential equation in (a).

4. Compute \( A^{-1} \) where
\[
A = \begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}.
\]

5. Solve the differential equation
\[
(y^2 + ty)dt - t^2dy = 0.
\]