1. Consider the following system of differential equations.

\[
\begin{align*}
  x' &= -x + 3y \\
y' &= -3x + 5y
\end{align*}
\]

(a) Write the above system in the form \( Y' = AY \) where \( A \) is a \( 2 \times 2 \) matrix and \( Y \) is a \( 2 \times 1 \) vector. \( \text{[2]} \)

(b) Find the eigenvalues and eigen vectors of the matrix \( A \). \( \text{[1+2]} \)

(c) Find a solution of the system \( Aq = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \). \( \text{[2]} \)

(d) Find two linearly independent solutions of the system \( Y' = AY \). \( \text{[3]} \)

2. Use Laplace transform methods to solve the system of differential equations

\[
\begin{align*}
x' &= 4x - 2y + 2u(t - 1) \\
y' &= 3x - y + u(t - 1)
\end{align*}
\]

subject to the initial conditions \( x(0) = 0, \ y(0) = \frac{1}{2} \). \( \text{[10]} \)

3. Solve the IVP, using the method of variation of parameters:

\[
4y'''' - y = te^{rac{1}{2}}, \quad y(0) = 1, y'(0) = 0.
\]

4. Find the general solution of \( 2y''' - 6y'' = t^2 \) using the method of underdetermined coefficients. \( \text{[7]} \)

5. Solve the homogeneous differential equation \( (y^2 + ty) \ dt + t^2 \ dy = 0 \). \( \text{[5]} \)

6. Solve the Bernoulli’s equation \( t^2 \frac{dy}{dt} - 2ty = 3y^4 \). \( \text{[5]} \)

7. Find the general solution of \( 16 \frac{d^4y}{dx^4} + 24 \frac{d^2y}{dx^2} + 9y = 0 \). \( \text{[5]} \)

8. Is the equation \( (ycosx + 2xe^y) \ dx + (sinx + x^2e^y - 1) \ dy = 0 \), exact? If it is exact, find the general solution of the equation. \( \text{[10]} \)