Answer all the questions. No credit will be given if only the answer is written without showing the relevant supporting work. Write legibly. The numbers at the end of each question indicate the maximum credit for the corresponding question.

1. Find the eigen values and eigenvectors of the matrix \[
\begin{pmatrix}
1 & 2 & 0 \\
0 & 1 & -2 \\
2 & 2 & -1
\end{pmatrix}.
\] [8]

2. Let \(A = \begin{pmatrix} 2 & 0 \\ 2 & 3 \end{pmatrix}\). Find the eigen values of \(A\) and the eigenvectors of \(A\) and \(A^T\). [4]

3. Use Cramer’s rule to find \(y\) without solving for \(x\) and \(z\)
\[
\begin{align*}
x + 2y + 3z &= -2 \\
3x - y + z &= 1 \\
-x + 4y - 2z &= -3
\end{align*}
\] [6]

4. The Augmented matrix for a system of linear equations is given by
\[
\begin{bmatrix}
1 & 1 & -3 & 2 & 1 \\
0 & 1 & 4 & 0 & 3 \\
0 & 0 & 0 & 1 & 2
\end{bmatrix}.
\] Find all solutions of the system. [4]

5. \(A = \begin{pmatrix} 1 & 0 \\ -5 & 2 \end{pmatrix}\)
   (a) Find elementary matrices \(E_1\) and \(E_2\) such that \(E_2E_1A = I\)
   (b) Write \(A^{-1}\) as a product of two elementary matrices.
   (c) Write \(A\) as a product of two elementary matrices. [3+2+3]