1. Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given reduced row echelon form. Solve the system. Assume that the variables are named $x_1, x_2, \ldots$, from left to right.

(i) $\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 7 \end{bmatrix}$  
(ii) $\begin{bmatrix} 1 & 0 & 0 & -7 & 8 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 1 & -5 \end{bmatrix}$  
(iii) $\begin{bmatrix} 1 & 2 & 0 & 2 & -1 & 3 \end{bmatrix}$

2. Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given row echelon form. Solve the system by reducing the matrix to reduced row echelon form. Assume that the variables are named $x_1, x_2, \ldots$, from left to right.

(i) $\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$  
(ii) $\begin{bmatrix} 1 & 7 & -2 & 0 & -8 & -3 \\ 0 & 0 & 1 & 1 & 6 & 5 \\ 0 & 0 & 0 & 1 & 3 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

3. Solve the linear system by Gauss-Jordan elimination:

\begin{align*}
&x_1 + x_2 + 2x_3 = 8 \\
&-x_1 - 2x_2 + 3x_3 = 1 \\
&3x_1 - 7x_2 + 4x_3 = 10
\end{align*}

(i) \begin{align*}
3x_1 + 2x_2 - x_3 &= -15 \\
5x_1 + 3x_2 + 2x_3 &= 0 \\
3x_1 + x_2 + 3x_3 &= 11 \\
6x_1 - 4x_2 + 2x_3 &= 30
\end{align*}

4. Solve the following linear homogeneous system:

\begin{align*}
2x_1 + x_2 + 3x_3 &= 0 \\
3x_1 + 2x_2 &= 0 \\
x_2 + 2x_3 &= 0
\end{align*}

(i) \begin{align*}
3x_1 + x_2 + x_3 + x_4 &= 0 \\
5x_1 - x_2 + x_3 - x_4 &= 0
\end{align*}

5. Determine the values of $a$ for which the system has no solutions, exactly one solution, or infinitely many solutions:

\begin{align*}
x + 2y + 3z &= 4 \\
3x - y + 5z &= 2 \\
4x + y - 14z &= a + 2
\end{align*}

6. What relationship must exist between $a, b, \text{ and } c$ for the following linear system to be consistent?

\begin{align*}
x + y + 2z &= a \\
x + z &= b \\
2x + y + 3z &= c
\end{align*}
7. Solve the two systems at once by row reduction:

\[
\begin{align*}
(i) & \quad x_1 + 2x_2 + x_3 = -1 \\
& \quad x_1 + 3x_2 + 2x_3 = 3 \\
& \quad x_2 + 2x_3 = 4
\end{align*}
\]

(ii) \quad x_1 + 3x_2 + 2x_3 = 0 \\
\quad x_2 + 2x_3 = 4

8. Find conditions on the b’s that ensure that the system is consistent:

\[
\begin{align*}
& x_1 - 2x_2 - x_3 = b_1 \\
& -2x_1 + 3x_2 + 2x_3 = b_2 \\
& -4x_1 + 7x_2 + 4x_3 = b_3
\end{align*}
\]

9. Determine the values of \( a \) for which the system has no solutions, exactly one solution, or infinitely many solutions.

(i) \quad x + \frac{2y}{2x + (a^2 - 5)y} = 1

10. Gauss method works by combining the equations in a system to make new equations. Can the equation \( 6x - 9y + 5z = -2 \) be derived, by a sequence of Gaussian reduction steps, from the following system?

\[
\begin{align*}
2x + y - z &= 4 \\
6x - 3y + z &= 5
\end{align*}
\]

(solution: yes; Find the steps!)

11. A box holding pennies, nickels and dimes contains thirteen coins with a total value of 83 cents. How many coins of each type are in the box? (Solution: 3 pennies, 4 nickels, 6 dimes)

12. For positive integers are given. Select any three of the integers, find their arithmetic average and add this result to the fourth integer. The numbers obtained are 29, 23, 21 and 17. What are the integers given. (Solution: 3, 9, 12 and 21)